Asteroseismic probing of low mass solar-like stars throughout their evolution with new techniques

COSPAR Conference 2022 – Athens

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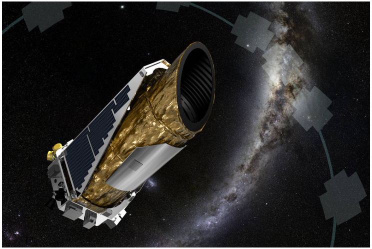
22^{nd} of June 2022

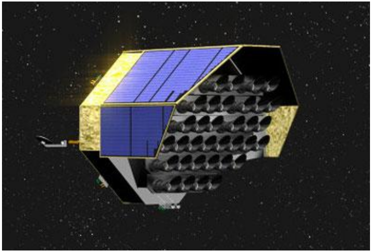
Large amount of data

Kepler (2009-2018)



Credits: NASA

PLATO (2026-. . .)



Credits: CNES

Several hundreds of thousands of pulsating stars!

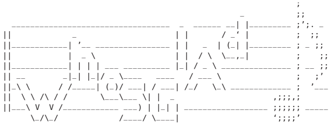
⇒ Unique opportunity for seismology: precise t, M, and R

Take advantage of the data

Large amount of very precise data!

$$\Downarrow$$

- Need for precise methods
  1. **WhoSGlAd**: Main-sequence stars (Farnir et al. 2019,2020)
  2. **EGGMiMoSA**: Sub- and red giants (Farnir et al. 2021)



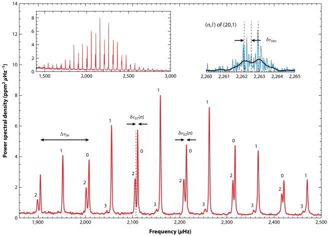
https://github.com/Yuglut/WhoSGlAd-python

Solar-like oscillation spectra

Smooth

$$\nu_{n,l} \simeq \left(n + \frac{l}{2} + \epsilon\right)\Delta\nu$$

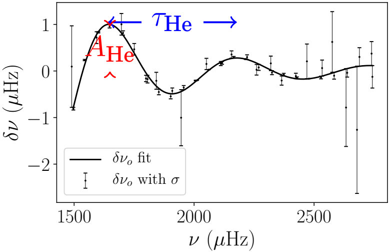
Tassoul (1980), Gough (1986)



Chaplin WJ, Miglio A. 2013.
Annu. Rev. Astron. Astrophys. 51:353–92

Glitches

$$\delta\nu = \nu_{\mathrm{obs}} - \nu_{\mathrm{smooth}}$$



WhoSGlAd: Principle

WhoSGlAd - **Who**le **S**pectrum and **Gl**itches **Ad**justment (Farnir et al. 2019,2020)

https://github.com/Yuglut/WhoSGlAd-python

Consider the frequencies vector space:

① Build orthonormal basis of functions (Gram-Schmidt);

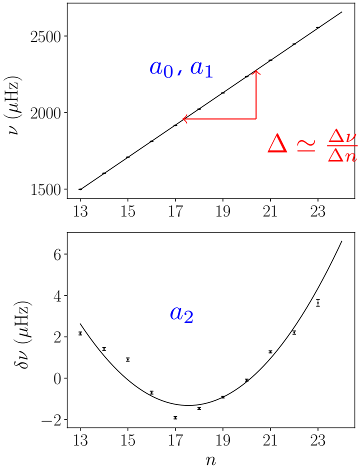
- From regular functions: $\boldsymbol{p}_k$
- Build orthonormal functions: $\boldsymbol{q}_k = \dfrac{\boldsymbol{p}_k - \sum_j^{k-1} \langle \boldsymbol{p}_k | \boldsymbol{q}_j \rangle \boldsymbol{q}_j}{\left\| \boldsymbol{p}_k - \sum_j^{k-1} \langle \boldsymbol{p}_k | \boldsymbol{q}_j \rangle \boldsymbol{q}_j \right\|}$
- With the scalar product: $\langle \boldsymbol{x} | \boldsymbol{y} \rangle = \sum_i^N \frac{x_i y_i}{\sigma_i^2}$

WhoSGIAd: Principle

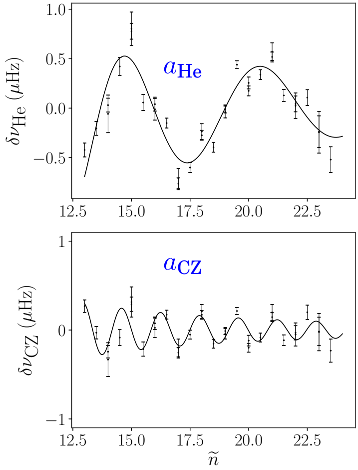
② Independent ν projections: $a_k = \langle \boldsymbol{\nu}_{\mathrm{obs}} | \boldsymbol{q}_k \rangle$

$$\Rightarrow \boldsymbol{\nu}_{\mathrm{fit}} = \sum_{k}^{K} a_k \boldsymbol{q}_k \;;$$

Smooth



Glitches



WhoSGlAd: Principle

③ Combine independent a_k into indicators as uncorrelated as possible;

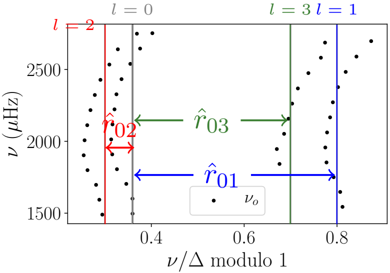
- $\Delta_l = a_{l,1} R^{-1}_{l,1,1}$,
- $\hat{r}_{0l} = \frac{a_{0,0} R^{-1}_{0,0,0} - a_{l,0} R^{-1}_{l,0,0}}{a_{0,1} R^{-1}_{0,1,1}} + \overline{\boldsymbol{n}_l} - \overline{\boldsymbol{n}_0} + \frac{l}{2}$,
- $\Delta_{0l} = \frac{a_{l,1} R^{-1}_{l,1,1}}{a_{0,1} R^{-1}_{0,1,1}} - 1$,
- $A_{\mathrm{He}} = \|\boldsymbol{\delta\nu}_{\mathrm{He}}\| = \sqrt{\sum a^2_{\mathrm{He}}}$,
- $\dots$

with $R^{-1}_{l,k,j}$ the transformation matrix: $\boldsymbol{q}_{l,k} = \sum_{j \leq k} R^{-1}_{l,k,j} \boldsymbol{p}_{l,j}$

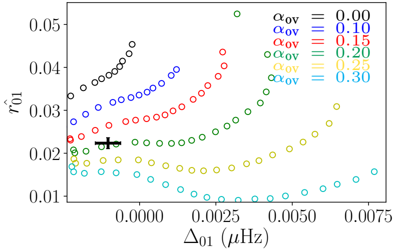
Seismic indicators

Smooth:

- $\hat{r}_{0l} \rightarrow$ Composition and evolution ($\sim$ Roxburgh & Vorontsov 2003)
- $\Delta_{0l} \rightarrow$ Overshooting (See also Deheuvels et al. 2016)



Farnir et al. (2019)

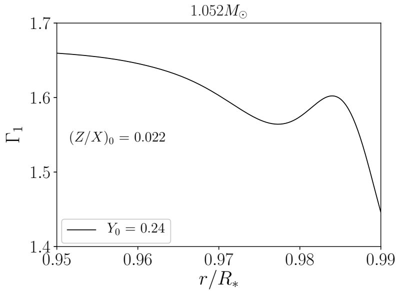


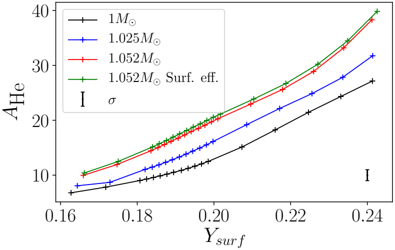
KIC 7206837

Glitch indicators

Glitch:

- $A_{\mathrm{He}} \rightarrow$ Helium content





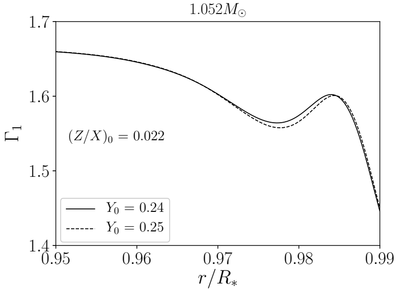
Farnir et al. (2019)

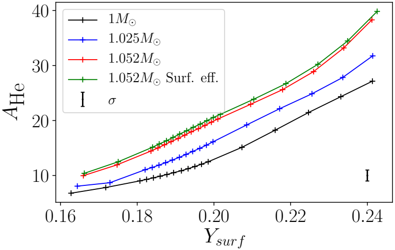
Independent of smooth indicators

Glitch indicators

Glitch:

- $A_{\mathrm{He}} \rightarrow$ Helium content





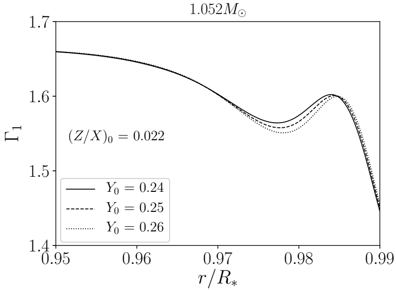
Farnir et al. (2019)

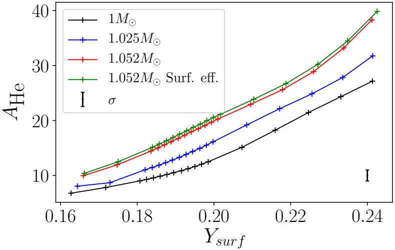
Independent of smooth indicators

Glitch indicators

Glitch:

- $A_{\mathrm{He}} \rightarrow$ Helium content





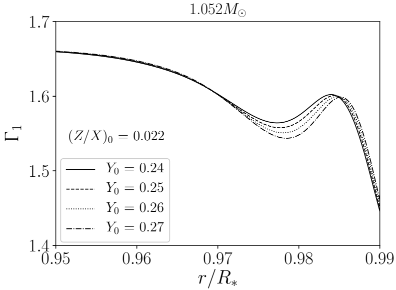
Farnir et al. (2019)

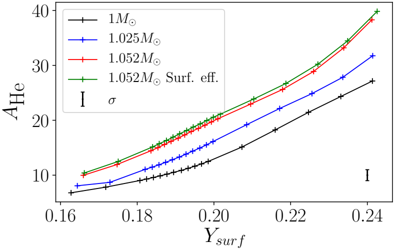
Independent of smooth indicators

Glitch indicators

Glitch:

- $A_{\mathrm{He}} \rightarrow$ Helium content



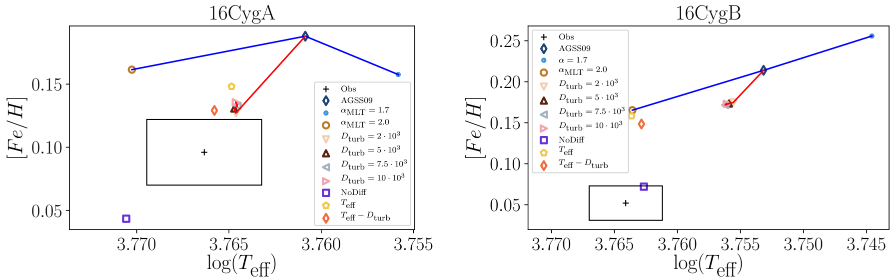


Farnir et al. (2019)

Independent of smooth indicators

Application to 16 Cygni

Fitting only Δ, $\hat{r}_{01}$, $\hat{r}_{02}$, and $A_{\mathbf{He}}$:



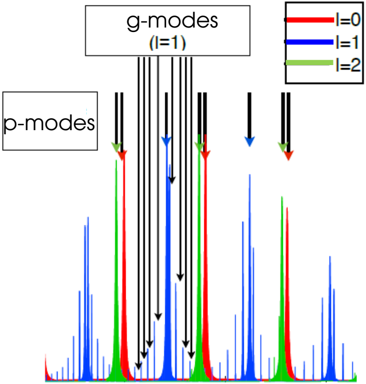
Seismology alone cannot discriminate models

(Farnir et al. 2020

See also Bulden et al. 2021)

Sub- and red giants: Mixed-Modes

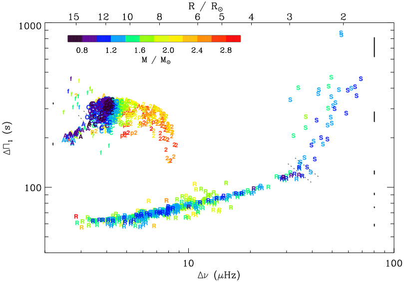
Pressure and **gravity** character
$\Rightarrow$ Probe the **whole** structure!



Credits: Grosjean (Thesis, 2015)

H-shell vs. **core-He** burning

(Montalbàn et al. 2010, Bedding et al. 2011)



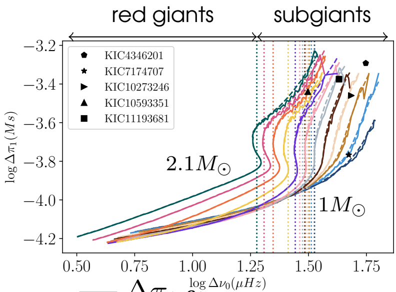
Credits: Mosser et al. (2014)

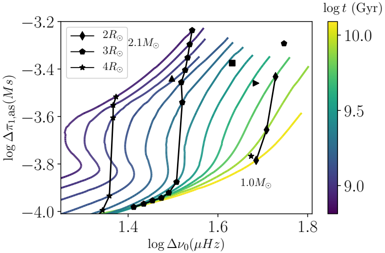
$\Delta\pi_1$: Period spacing

EGGMiMoSA

EGGMiMoSA:
Extracting **G**uesses about **G**iants via **Mi**xed-**Mo**des **S**pectrum **A**djustment (Farnir et al. 2021)

Info on **mass**, **radius**, and **age**





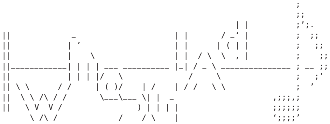
— $\Delta\pi_{1,\mathrm{fit}}$

$$- - -\ \Delta\pi_{1,\mathrm{asy}} = 2\pi^2\left(\int_g \frac{N}{r}dr\right)^{-1}$$

Farnir et al. (2021)

Conclusions

- Two methods to probe most of the evolution of solar-like pulsators;
- Fast ($< 1s$ per star) and automated;
- Robust indicators for stellar modelling;
- Well suited candidates for the analysis of the PLATO data.



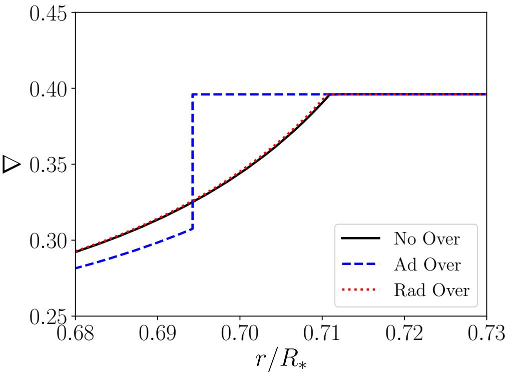
https://github.com/Yuglut/WhoSGlAd-python

Appendices

Convection Zone Glitches

Mixing processes badly constrained

→ Convection zone glitch : radiative - convective regions transition $\Rightarrow$ Transition localisation



WhoSGlAd: Basis Elements

We selected the basis functions:

- Smooth
  ① $p_0(n) = 1$
  ② $p_1(n) = n$
  ③ $p_2(n) = n^2$
- Glitch
  He $$\begin{aligned} p_{\mathrm{He}Ck}(\widetilde{n}) &= \cos\left(4\pi T_{\mathrm{He}}\widetilde{n}\right)\widetilde{n}^{-k} \\ p_{\mathrm{He}Sk}(\widetilde{n}) &= \sin\left(4\pi T_{\mathrm{He}}\widetilde{n}\right)\widetilde{n}^{-k} \end{aligned}$$
  with $k = 5, 4$, $\widetilde{n} = n + l/2$
  CZ $$\begin{aligned} p_{\mathrm{C}C}(\widetilde{n}) &= \cos\left(4\pi T_{\mathrm{CZ}}\widetilde{n}\right)\widetilde{n}^{-2} \\ p_{\mathrm{C}S}(\widetilde{n}) &= \sin\left(4\pi T_{\mathrm{CZ}}\widetilde{n}\right)\widetilde{n}^{-2} \end{aligned}$$

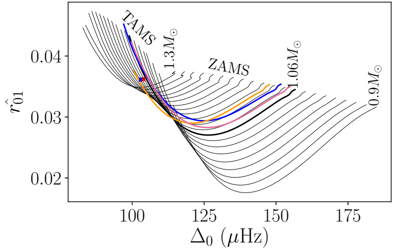
WhoSGlAd: $\hat{r}_{01}$

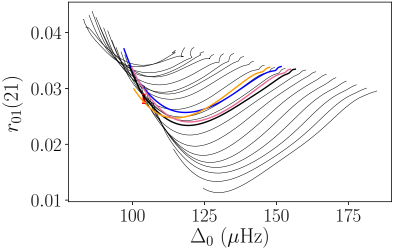
WhoSGlAd

$$\hat{r}_{01} = \frac{\overline{\nu_0} - \overline{\nu_1}}{\Delta_0} + \overline{n_1} - \overline{n_0} + \frac{1}{2}$$

Roxburgh & Vorontsov (2003)

$$r_{01}(n) = \frac{\nu_{n-1,1} - 2\nu_{n,0} + \nu_{n,1}}{2(\nu_{n,1} - \nu_{n-1,1})}$$





16 Cyg A : $\Delta\hat{r}_{01}/\hat{r}_{01} = 0.7\%$

$\Delta r_{01}(21)/r_{01}(21) = 2.9\%$

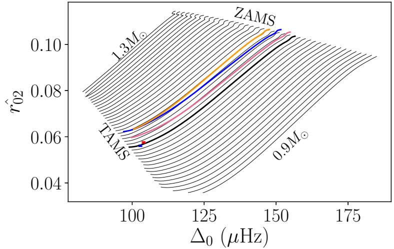
$(Z/X)_0 = 0.0218$ $\alpha_{\mathrm{MLT}} = 1.82$ $Y_0 = 0.25$

$(Z/X)_0 = 0.018$ $\alpha_{\mathrm{MLT}} = 1.5$ $Y_0 = 0.27$

WhoSGIAd: $\hat{r}_{02}$

WhoSGIAd

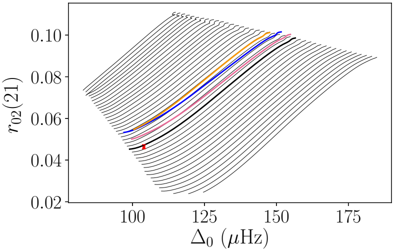
$$\hat{r}_{02} = \frac{\overline{\nu_0} - \overline{\nu_2}}{\Delta_0} + \overline{n_2} - \overline{n_0} + \frac{2}{2}$$



16 Cyg A : $\Delta\hat{r}_{02}/\hat{r}_{02} = 0.6\%$

Roxburgh & Vorontsov (2003)

$$r_{02}(n) = \frac{\nu_{n,0} - \nu_{n-1,2}}{(\nu_{n,1} - \nu_{n-1,1})}$$

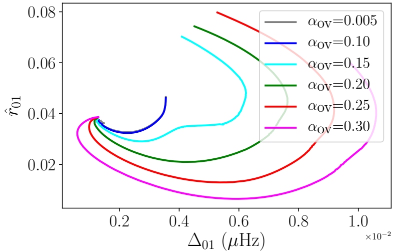


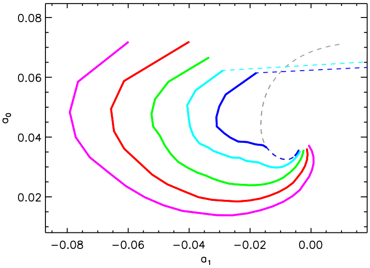
$\Delta r_{02}(21)/r_{02}(21) = 2.1\%$

$(Z/X)_0 = 0.0218$ $\alpha_{\mathrm{MLT}} = 1.82$ $Y_0 = 0.25$

$(Z/X)_0 = 0.018$ $\alpha_{\mathrm{MLT}} = 1.5$ $Y_0 = 0.27$

WhoSGlAd: Δ_{0l} & Overshooting



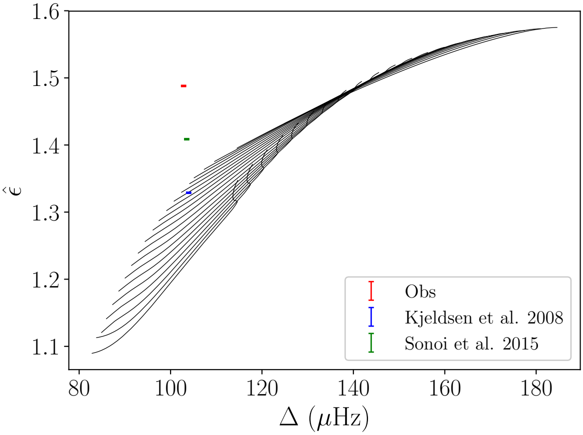


Credits: Deheuvels et al. 2016

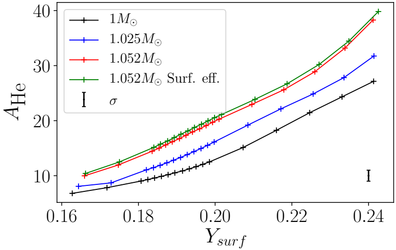
- $\hat{r}_{01}$: mean $r_{01}(n)$
- Δ_{01}: slope in n of $r_{01}(n)$

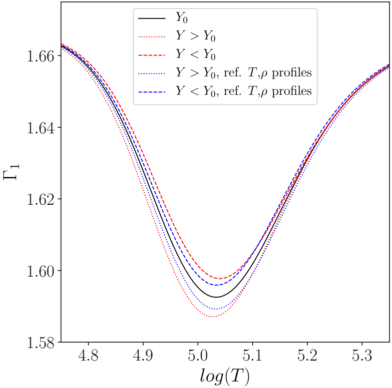
- a_0: mean $r_{01}(n)$
- a_1: slope in n of $r_{01}(n)$

WhoSGlAd: ϵ and surface effects

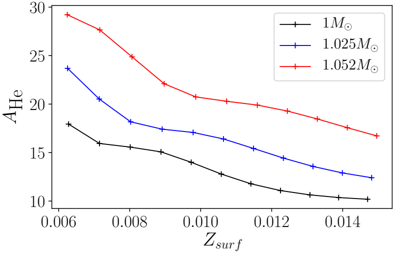


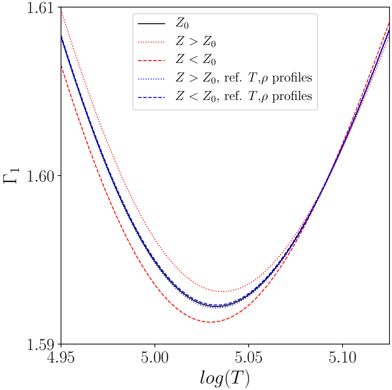
WhoSGIAd: Helium and Γ_1 toy model





WhoSGlAd: Metallicity and Γ_1 toy model

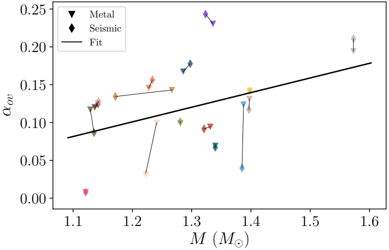


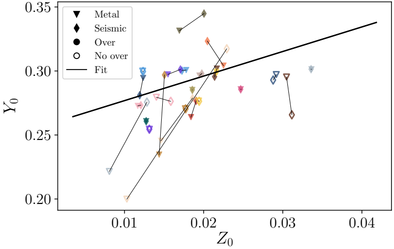


WhoSGlAd: Application to the Kepler LEGACY sample

- Overshooting
  $\Delta\alpha_{\mathbf{ov}}/\Delta M = 0.2 \pm 0.1$,
  $\alpha_{\mathbf{ov},0} = -0.1 \pm 0.2$

- Galactic enrichment
  $\Delta Y/\Delta Z = 1.92 \pm 0.79$,
  $Y_p = 0.26 \pm 0.01$





Free param.: t, M, X_0, $(Z/X)_0$, and $\alpha_{\mathbf{ov}}$;

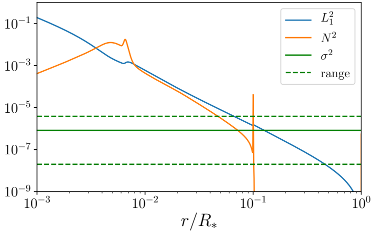
Seismic: Models with only Δ, $\hat{r}_{01}$, $\hat{r}_{02}$, Δ_{01}, and $A_{\mathbf{He}}$;

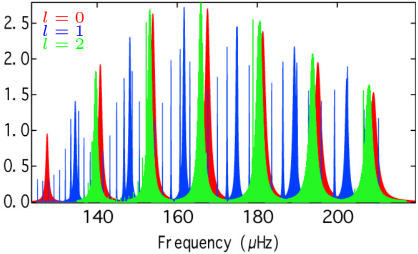
Metal: Models with only Δ, $\hat{r}_{01}$, $\hat{r}_{02}$, Δ_{01}, and $[\mathrm{Fe/H}]$.

Mixed-modes

- Modes of mixed p and g character

→ pressure and gravity cavities coupled via evanescent region





Credits: Grosjean et al. (2014)

EGGMiMoSA: Formalism

EGGMiMoSA:
Extracting **G**uesses about **G**iants via **Mi**xed-**Mo**des **S**pectrum **A**djustment (Farnir et al. 2021)

Asymptotic coupling between p- and g-cavity:

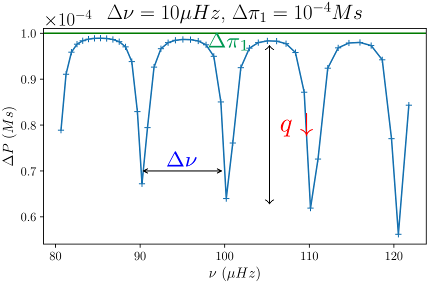
$$\tan\theta_p = q\tan\theta_g$$

where:

$$\theta_p = \pi\left[\frac{\nu}{\Delta\nu} - \epsilon_p\right]$$

$$\theta_g = \pi\left[\frac{1}{\nu\Delta\pi_1} - \epsilon_g + \frac{1}{2}\right]$$

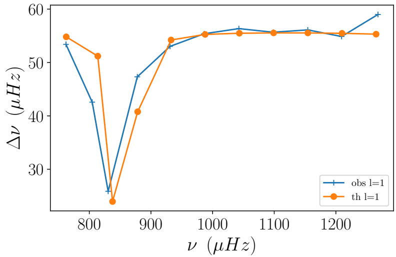
Shibahashi (1979),
Mosser et al. (2015)



5 parameters L-M minimisation: $\Delta\nu$, $\Delta\pi_1$, ϵ_p, ϵ_g, q
No further simplifications ⇒ adapted to red and subgiants

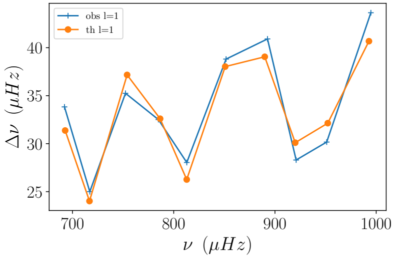
EGGMiMoSA: Fit examples

KIC4346201



$$M \sim [1.2M_\odot, 1.3M_\odot]$$
$$R \sim 2R_\odot$$
$$\log t \sim 9.5$$

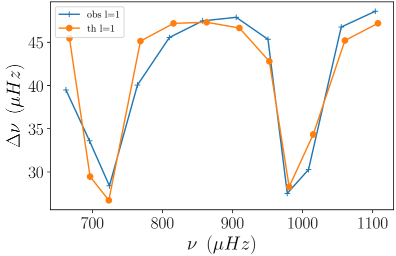
KIC7174707



$$M \sim 1.1M_\odot$$
$$R \sim 2R_\odot$$
$$\log t \sim 9.8$$

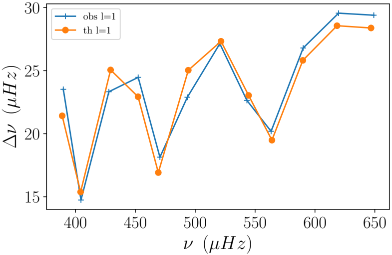
EGGMiMoSA: Fit examples

KIC10273246



$$M \sim 1.3M_\odot$$
$$R \sim 2.1R_\odot$$
$$\log t \sim 9.7$$

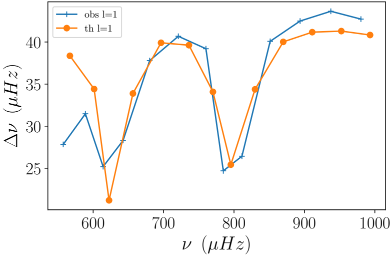
KIC10593351



$$M \sim 1.9M_\odot$$
$$R \sim 3R_\odot$$
$$\log t \sim 9$$

EGGMiMoSA: Fit examples

KIC11193681



$$\begin{aligned} M \sim& \quad 1.5M_\odot \\ R \sim& \quad 2.5R_\odot \\ \log t \sim& \quad 9.5 \end{aligned}$$