

Towards an effective asteroseismology of solar-like stars: time-dependent convection effects on pulsation frequencies

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ABSTRACT

Since the early days of helioseismology, adiabatic models have shown their limits for a precise fitting of individual oscillation frequencies. This discrepancy, which also exists for solar-type stars, is known to originate near the surface superadiabatic convective region where the interaction between oscillations and convection is likely to have a large effect on the frequencies. We present an asteroseismic study to address the adequacy of time-dependent convection (TDC) non-adiabatic models to better reproduce the observed individual frequencies. We select, for this purpose, three solar-like stars, in addition to the Sun, to which we fit the observed frequencies in a grid of TDC non-adiabatic models. The best model selection is done by applying a maximum likelihood method. The results are compared to pure adiabatic and near-surface corrected adiabatic models. We show that, first, TDC models give very good agreement for the mode frequencies and average lifetimes. In the solar case, the frequency discrepancy is reduced to $<1.75 \mu\text{Hz}$ over 95 per cent of the modes considered. Secondly, TDC models give an asteroseismic insight into the usually unconstrained ad hoc stellar parameters, such as the mixing-length parameter α_{MLT} .

Key words: asteroseismology – convection – Sun: oscillations – stars: oscillations.

1 INTRODUCTION

A long outstanding problem in solar oscillations is the offset between the theoretical and observed frequencies (e.g. Christensen-Dalsgaard 1988; Dziembowski, Paterno & Ventura 1988). Here discrepancy has the following properties: (1) it is spherical degree ℓ -independent; (2) it is radial order n -dependent, i.e. it is small for the low-order radial modes and increases significantly for modes with frequencies close to the cut-off frequency of the solar photosphere (~ 5 mHz). This indicates that this offset originates in layers to which the low-order modes practically do not penetrate but where the high-order modes have a significant amplitude. Consequently, the origin of the discrepancy lies in the outer layers of the Sun. Similar problem has been found for solar-like stars, since they pulsate in the same regime and are likely excited by the same mechanism.

Several methods have been proposed to remove these surface effects, e.g. by considering the ratio between the small and large separations (Roxburgh & Vorontsov 2003). Kjeldsen, Bedding &

Christensen-Dalsgaard (2008) proposed to cancel this effects by introducing an empirical near-surface correction calibrated to the Sun to rescale the adiabatic frequencies to better compare model frequencies with the observations.

There are mainly two complementary approaches to address this problem, either by studying the internal structure effects or by studying the physics of the oscillation effects on the frequencies. The first one, called ‘model effects’, consists of improvements in the physics of the solar model and investigates the effects on the p mode frequencies due to equation of state, atmosphere structure, treatment of the convection, etc. (e.g. Christensen-Dalsgaard, Dappen & Lebreton 1988; Cox, Guzik & Kidman 1989; Balmforth & Gough 1990; Guenther 1994; Gabriel 1996a; Gabriel & Carlier 1997; Li et al. 2002). The ‘modal effects’, which is the second approach, is the study of the coupling of the oscillations to the convection using time-dependent convection (TDC) theory, for which different formulations have been proposed (Gough 1977; Stellingwerf 1982; Balmforth 1992a,b; Gabriel 1996b; Xiong, Cheng & Deng 1997). On the other hand, numerical simulations of convection in the solar surface layers (e.g. Stein & Nordlund 1991, 1994) shed a new light on the complex phenomena at

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Table 1. Fundamental parameters of the selected stars.

Star	T_{eff} (K)	$\log g$	$\log(L/L_{\odot})$	[Fe/H]	M (M_{\odot})	R (R_{\odot})	Age (Gyr)	$\langle \Delta \nu_{\text{obs}} \rangle$ (μHz)	$\langle \delta \nu_{\text{obs}} \rangle$ (μHz)	Ref.
Sun	5778 ± 10	4.44 ± 0.01	0.0	0.00 ± 0.01	1.00	1.00	4.57	136.05	9.90	1, 2
α Cen A	5832 ± 62	4.33 ± 0.11	0.18 ± 0.02	0.23 ± 0.05	1.105 ± 0.007	1.224 ± 0.003	–	106.28	6.12	3, 4, 5
β Hyi	5860 ± 63	4.04 ± 0.10	0.55 ± 0.02	-0.07 ± 0.05	–	1.809 ± 0.015	–	57.48	5.11	3, 5, 6
τ Cet	5310 ± 62	4.44 ± 0.10	-0.31 ± 0.02	-0.52 ± 0.03	–	0.793 ± 0.004	–	169.60	12.70	7, 8

References: (1) Gray (1992); (2) Bahcall, Pinsonneault & Wasserburg (1995); (3) This work; (4) Kervella et al. (2003); (5) Bruntt et al. (2010); (6) Kjeldsen et al. (2005); (7) Teixeira et al. (2009); (8) Sousa et al. (2008).

play in these layers. Rosenthal et al. (1999) used these numerical simulations to study the effects of turbulent convection on solar frequencies.

Furthermore, in convective zones, stars lose their radial symmetry and static state. All variables show large fluctuations over small horizontal scales (Nordlund 1985; Steffen, Ludwig & Kruess 1989). Therefore, the near-surface solar layers not only are the spatial location where the solar p modes have their upper turning points and are excited and damped, but also show large amplitude fluctuations due to convective motions, which will modify the medium properties; in that case, the total pressure is the sum of the gas pressure and the turbulent pressure. Consequently, the oscillation waves crossing these layers see them move and their properties varying with time. We are far from a propagation in a static homogeneous isotropic medium. If in addition to that we remember that the characteristic time of convection is of the same order as the pulsation periods in the outer convection envelope, this leads to the conclusion that any reliable study must include a time-dependent theory of convection (Gabriel 2000).

In the framework of mixing-length theory, Gabriel (1996b) proposed the only TDC formalism pertinent to non-radial modes. This treatment, given with details in Grigahcène et al. (2005), has been implemented in the non-radial non-adiabatic code MAD (Dupret 2001). It gives a noticeable improvement in modelling solar-like oscillations, which can be appreciated especially in the calculation of the following. (1) The frequencies, where a shift is found. This offset is due to dissipation and increases near the cut-off frequency. (2) The mode lifetimes (inverses of the damping rates) directly related to the line widths. TDC models are required to compute the damping rates, since the damping of solar-like p modes results from the coherent interaction between convection and oscillations (Balmforth 1992a; Dupret et al. 2006).

Although they have their own uncertainties, non-adiabatic TDC models include a physical treatment of the superficial layers of stars taking the convection–oscillation interaction into account. Thus, they are a more robust tool to precisely exploit the full information given by pulsation frequencies. We propose to extend the application of this treatment to the case of high-order solar-like oscillations.

In addition to the Sun, we select three solar-like stars in order to study them using TDC models. We compare the results with adiabatic oscillations which include, or do not include, the empirical near-surface correction proposed by Kjeldsen et al. (2008). Detailed discussion of the asteroseismic determination of chemical composition, age, etc. is postponed to a forthcoming work.

This Letter is divided as follows. In Section 2, we present the selected objects under study. Theoretical model fitting using individual oscillation frequencies is considered in Section 3. In Section 4, we discuss the results and the main implications for the seismic study of solar-like stars.

2 THE SELECTED STARS

In this work, we present the results for the Sun together with three of its analogues. All of them show solar-like oscillations.

(i) Sun, by far the star we know best. It shows oscillations observed continuously, e.g., by the Birmingham Solar Oscillations Network (BiSON) for several decades now. In this work we use the corrected frequencies suitable to the quiet Sun given in table 2 of Broomhall et al. (2009).

(ii) α Centauri A (α Cen A), the primary component of a triple stellar system which is a G2 V main-sequence star. We use the frequency list given by Bazot et al. (2007).

(iii) β Hydri (β Hyi) is a G2 IV subgiant star. Brandão et al. (2011) give an updated list of its pulsation frequencies with a detailed asteroseismic study as well.

(iv) τ Ceti (τ Cet) is a G8 V main-sequence star. Teixeira et al. (2009) give a list of its pulsation frequencies.

Some of the observational parameters of these stars are summarized in Table 1. For α Cen A and β Hyi, the spectra were obtained from the ESO archive data. In total we have downloaded 20 spectra, for each star, that were observed with HARPS at the La Silla ESO 3.6-m Telescope with a resolution $R \sim 110\,000$. The spectra were then combined using IRAF tools. For β Hyi we have combined all the 20 downloaded spectra to obtain a final spectrum with $S/N \sim 1500$. For α Cen A, only 16 spectra were necessary to reach a final spectrum with $S/N \sim 2000$. The spectroscopic analysis of the stars has been conducted on the combined HARPS spectra, using ARES (Sousa et al. 2007) and following the method described in Sousa et al. (2008). The values of T_{eff} , $\log g$ and [Fe/H] were derived, and the actual errors on these parameters were derived following the discussion presented in Sousa et al. (2011).

The equilibrium stellar models have been computed using the Code Liégeois d'Évolution Stellaire (CLEM; Scuflaire et al. 2008), where the input physics included is similar to the one used in Dupret et al. (2005).

3 RESULTS

We search the best model in a grid of theoretical models by requiring them to fit the observed frequencies within the quoted observational uncertainties. A maximum likelihood method is used:

$$\mathcal{L} = \left(\prod_i \frac{1}{\sqrt{2\pi}\sigma_i} \right) \exp(-\chi^2/2),$$

where $\chi^2 = \chi_v^2 + \chi_{\odot}^2$. The first part quantifies the closeness of the theoretical frequencies ν^{theo} to the observed frequencies ν^{obs} within

Table 2. Main properties of the best selected models.

Star	Model	M (M_{\odot})	R (R_{\odot})	Age (Gyr)	T_{eff} (K)	$\log g$	$\log(L/L_{\odot})$	Z/X	α_{MLT}	α_{Ov}	$\langle \Delta \nu \rangle$ (μHz)	$\langle \delta \nu \rangle$ (μHz)	χ^2_{ν}	$\chi^2_{\mathcal{O}}$	(a, r)
Sun	Adiab.	1.0000	1.0000	4.5889	5780	4.4380	0.0008	0.0251	1.810	0.00	136.79	10.10	34 433.75	0.02	–
	Corr.	1.0000	1.0000	4.5774	5779	4.4383	0.0004	0.0251	1.810	0.00	135.86	10.05	13 361.54	0.01	(–5.00, 1.002564)
	TDC	1.0000	1.0000	4.5774	5779	4.4383	0.0004	0.0251	1.810	0.00	135.86	10.09	2838.99	0.01	–
α Cen A	Adiab.	1.1041	1.2225	6.4143	5784	4.3068	0.1762	0.0444	1.802	0.25	106.58	4.85	3.95	0.37	–
	Corr.	1.1065	1.2206	5.7018	5821	4.3091	0.1861	0.0441	1.775	0.30	105.94	5.70	0.57	0.37	(–3.76, 1.000019)
	TDC	1.1065	1.2204	5.7505	5826	4.3093	0.1874	0.0442	1.775	0.36	105.74	5.98	0.49	0.34	–
β Hyi	Adiab.	1.0700	1.8223	8.2923	5856	3.9464	0.5445	0.0194	2.500	0.00	57.67	5.87	0.58	0.29	–
	Corr.	1.0800	1.8207	7.8221	5856	3.9513	0.5436	0.0194	1.900	0.00	57.63	5.46	0.40	0.31	(–2.14, 0.999946)
	TDC	1.0720	1.8211	8.1957	5858	3.9478	0.5446	0.0194	2.300	0.00	57.43	5.52	0.14	0.26	–
τ Cet	Adiab.	0.7850	0.7935	7.2399	5371	4.5341	–0.3278	0.0074	1.250	0.09	170.48	9.64	14.336	0.44	–
	Corr.	0.7890	0.7935	6.8387	5368	4.5363	–0.3286	0.0074	1.240	0.09	169.34	9.81	3.010	0.45	(–12.51, 1.000562)
	TDC	0.7885	0.7936	6.9157	5368	4.5358	–0.3286	0.0074	1.240	0.09	169.44	9.88	3.753	0.44	–

the observed uncertainty σ_i^{ν} ; it is given by

$$\chi_{\nu}^2 = \frac{1}{N_{\nu}} \sum_{i=1}^{N_{\nu}} \left(\frac{\nu_i^{\text{theo}} - \nu_i^{\text{obs}}}{\sigma_i^{\nu}} \right)^2.$$

The second part determines the closeness of the $N_{\mathcal{O}}$ global parameters of the theoretical model $\mathcal{O}^{\text{theo}}$ to the observed values \mathcal{O}^{obs} , namely the effective temperature T_{eff} , the mass M , the gravity $\log g$, the radius R and the chemical composition (X, Z):

$$\chi_{\mathcal{O}}^2 = \frac{1}{N_{\mathcal{O}}} \sum_{j=1}^{N_{\mathcal{O}}} \left(\frac{\mathcal{O}_j^{\text{theo}} - \mathcal{O}_j^{\text{obs}}}{\sigma_j^{\mathcal{O}}} \right)^2.$$

The solar age (Bahcall et al. 1995) is taken into account as well. We restrict the calculation to the N_{ν} observed radial modes to avoid any complication that might arise by the rotational splitting which could affect $\ell \geq 1$ modes.

For comparison, we also consider the near-surface correction by following the prescription of Kjeldsen et al. (2008) and Brandão et al. (2011). Particularly, we use the χ_{ν}^2 formula as given in equation (10) of Brandão et al. (2011) to search for the best corrected model. We adopt the value 4.90 for the coefficient b , as suggested by Kjeldsen et al. (2008), and for the Sun we adopt $a = -5$.

Our methodology applies the Sun as star approach. In this case, because of the very high accuracy of the frequencies, an extra weight is given to $\chi_{\mathcal{O}}^2$ to avoid that the global χ^2 is mainly driven by χ_{ν}^2 .

Table 2 lists the properties of the best models. Also given are the average small and large separations together with the quantities χ_{ν}^2 and $\chi_{\mathcal{O}}^2$. The coefficients (r, a) are related to the near-surface correction following Kjeldsen et al. (2008) notation. The reference frequency ν_0 required by the correction is given in Table 3.

Table 3. Average TDC theoretical lifetimes associated with the oscillation modes for the selected stars. Observational data are taken from Chaplin et al. (2009).

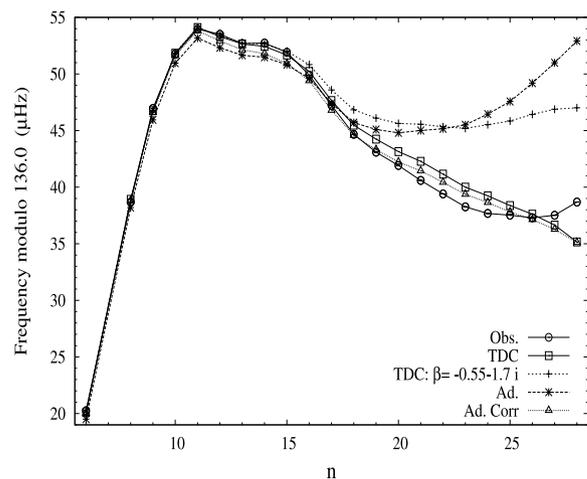
Star	ν_0 (μHz)	$\langle \tau_{\text{obs}} \rangle$ (d)	$\langle \tau_{\text{TDC}} \rangle$ (d)	ν (μHz)
Sun	3000	3.16 ± 0.2	3.17	$[2200, 4000]_{\ell=0,1}$
α Cen A	2410	3.9 ± 1.4	3.64	$[1900, 2900]_{\ell=0}$
β Hyi	1000	$2.3^{+0.6}_{-0.5}$	2.28	$[900, 1200]_{\ell=0}$
τ Cet	4500	1.7 ± 0.5	1.75	$[3800, 5200]_{\ell=0}$

3.1 Mode frequencies and lifetimes

The comparison between theoretical and observed frequencies is shown in Figs 1–4 and the values of χ_{ν}^2 are given in column 14 of Table 2. The very large value of χ_{ν}^2 for the Sun is due to the very small values of the uncertainties ($\sigma^{\nu} \leq 0.323 \mu\text{Hz}$) associated with its frequencies.

At the outset, we note that for the four cases, adiabatic models have the worst agreement with the observations. As can be clearly seen in Figs 1 and 4, and at some level in Fig. 2, the discrepancy between adiabatic and observed frequencies is most notable for intermediate and high-order modes. The TDC frequencies are the closest to the observed ones, as shown by the values of χ_{ν}^2 . This is better seen in the solar case where TDC χ_{ν}^2 is one order of magnitude lower than the adiabatic one, which is illustrated in Fig. 1. For τ Cet, TDC and near-surface corrected models give comparable χ_{ν}^2 (Fig. 4). A new and hopefully successful observational campaign is highly recommended for τ Cet, which is one of the lowest mass known solar-like stars.

Since the near-surface correction aims at forcing the theoretical frequencies to match the observations, it significantly reduces the discrepancy but does not cancel it totally yet. In the case of β Hyi,


Figure 1. Solar radial mode frequencies modulo the large separation versus the radial order (n). Crosses and triangles, respectively, represent adiabatic and corrected adiabatic frequencies, while circles and squares show observed and TDC frequencies, respectively. Parameters of the calibrated model are listed in Table 2.

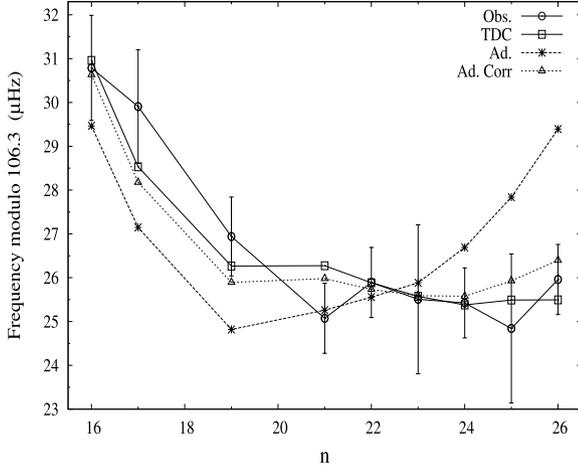


Figure 2. The same as in Fig. 1, but for α Cen A radial modes.

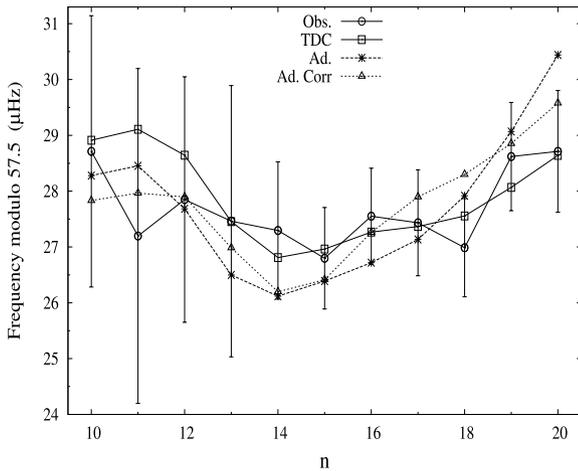


Figure 3. The same as in Fig. 1, but for β Hyi radial modes.

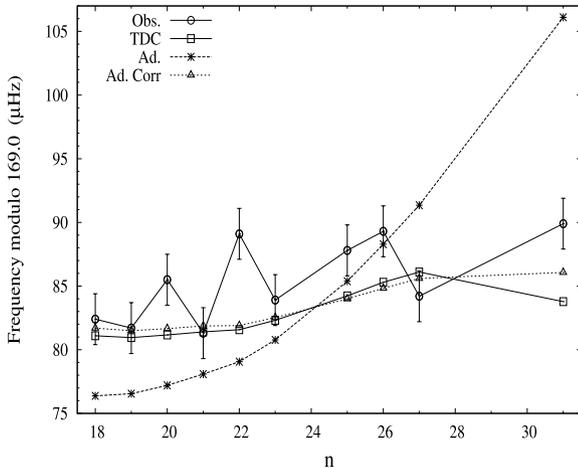


Figure 4. The same as in Fig. 1, but for τ Cet radial modes.

the near-surface correction seems to work the worse compared to the other cases. For α Cen A, TDC χ^2_V is of the same order as in corrected adiabatic model. This might indicate the limitation of the validity of the correction to the vicinity of the sun for which it has been calibrated.

The TDC formalism, as implemented, allows the study of the influence of different perturbative terms. Dupret et al. (2005) showed that these different terms, taken into account separately or in combination, have small influence on the value of the oscillation frequencies for low-order modes. This is definitely not the case for high-order mode frequencies. We note that for α Cen A and β Hyi, the calculations include the perturbation of the convective flux and the turbulent pressure, while for the Sun and τ Cet they include only the perturbation of the convective flux.

When the perturbation of the turbulent pressure is required, its equilibrium value is estimated a posteriori and included in the model without disturbing the hydrostatic equilibrium at the price of a small inconsistency with the equation of state. Estimation of the turbulent pressure in solar models indicates that it does not exceed at most 15 per cent of the total pressure (Houdek 2010).

A known problem in local TDC treatment is the short-wavelength spatial oscillations of the eigenfunctions. In order to remove them, Grigahcène et al. (2005) introduced a different perturbation of the closure equations via a complex parameter β . Plus symbols in Fig. 1 give the result for ($\beta = -0.55 - 1.7i$), which yields the better fit of the observed solar mode lifetimes (Belkacem et al. 2011). The difference between this treatment and the pure TDC one indicates that the frequencies are affected by the used perturbation of closure equations.

Another aspect we would like to stress is the capability of our models to calculate the theoretical lifetime of the oscillation modes. Average TDC lifetimes ($\langle \tau_{\text{TDC}} \rangle$) are given in Table 3 together with the observed ones ($\langle \tau_{\text{obs}} \rangle$) (Chaplin et al. 2009). Our TDC models provide a very encouraging result for all stars even though individual mode lifetimes are missing for the stars other than the Sun.

3.2 Stellar global and ad hoc parameters

Columns 3–9 in Table 2 give the global parameter obtained for the best models, while column 15 gives the values of χ^2_O . The selected models are all within the photometric observational boxes. Adiabatic models give the highest values for the radii of α Cen A and β Hyi; however, they give the lowest estimation of the masses. TDC and near-surface corrected models give the same value for α Cen A mass, but they give different mass value for τ Cet and β Hyi. Near-surface correction tends to give the lowest estimation of the age, which can be explained by the associated values of the mixing-length parameters α_{MLT} . Pure adiabatic models always give the highest age values associated with the highest values of α_{MLT} .

Columns 12 and 13 in Table 2 give the average large and small separations, respectively. The obtained values are very close to each other. They all differ by $< 1 \mu\text{Hz}$ when compared to the observed ones in general (with the exception of the adiabatic $\langle \delta\nu \rangle$ in α Cen A). Although in terms of relative difference, it is less than 1 per cent, it does have an important impact in the determination of the stellar ages and radii (Houdek 2010), as can be seen in Table 2.

On the other hand, the results show very different values for the stellar evolution ad hoc parameters α_{MLT} and α_{OV} , respectively. The value of α_{MLT} for α Cen A is the closest to the solar one, while it is higher for β Hyi and lower for τ Cet, in agreement with the expected variability of the MLT parameter on the HR diagram (Trampedach & Stein 2011).

4 CONCLUSIONS

In this work we have considered how a physically more robust representation of the mode physics near the surface can provide

a better fitting of the stellar models, based on seismic data. To do so, we used a χ^2 criterion, based on seismic and non-seismic parameters, to provide an indication of the best solution when we compare the models with the observations. The main results we found can be summarized as the following.

(i) Frequencies of solar-like oscillations calculated using TDC provide a significant improvement compared with the adiabatic frequencies. However, our TDC models do not succeed yet in matching at the same time all frequencies. Differences between theoretical model frequencies and observations are still significant (above the observational uncertainties) for higher frequency modes, especially for the solar case. Further improvements on the theoretical description of how the oscillations interact with convection are still required to fully reproduce the solar data.

(ii) By comparing the best model fitting obtained using TDC frequencies and adiabatic frequencies corrected by the surface effect according to Kjeldsen et al. (2008), it is shown that the first achieves smaller frequency differences and closer average frequency separations, resulting in different stellar ages.

(iii) The values of the mode lifetime obtained with TDC is within 1σ of the observed values, confirming the better physical description of the mode physics near the surface provided by TDC when applied to solar-like stars.

In conclusion, TDC provides an improvement on the description of the mode physics near the surface of solar-like stars, it being a more robust base for model fitting of stars with convective envelopes using precise seismic data. Consequently, it may thus be very important to use TDC models for any detailed asteroseismic study of solar-like stars and their near-surface structure, including a possible calibration of convection through the mixing length.

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