



# Theoretical Aspects of g-mode Pulsations in $\gamma$ Doradus Stars

M.-A. Dupret<sup>1</sup>, A. Grigahcène<sup>2,3</sup>, R. Garrido<sup>2</sup>, J. De Ridder<sup>5</sup>, A. Moya<sup>1</sup>, J.-C. Suárez<sup>2</sup>,  
R. Scuflaire<sup>4</sup>, M. Gabriel<sup>4</sup>, and M.-J. Goupil<sup>1</sup>

<sup>1</sup> Observatoire de Paris, LESIA, CNRS UMR 8109, 92195 Meudon, France

<sup>2</sup> Instituto de Astrofísica de Andalucía-CSIC, Apartado 3004, 18080 Granada, Spain

<sup>3</sup> CRAAG - Algiers Observatory BP 63 Bouzareah 16340, Algiers, Algeria

<sup>4</sup> Institut d'Astrophysique et de Géophysique de l'Université de Liège, Belgium

<sup>5</sup> Instituut voor Sterrenkunde, Katholieke Universiteit Leuven, 3001 Leuven, Belgium  
e-mail: MA.Dupret@obspm.fr

**Abstract.**  $\gamma$  Dor stars are main sequence variable A-F stars whose long periods (between 0.35 and 3 days) correspond to high-order gravity modes pulsation. Most of them are multi-periodic. We will concentrate here on two theoretical aspects of these stars. First, an analysis of the driving mechanism of the  $\gamma$  Dor g-modes is presented, using the linear Time-Dependent Convection (TDC) treatment of Gabriel (1996) and Grigahcène et al. (2005). This driving is due to a periodic flux blocking mechanism at the base of their convective envelope. The location of the blue and red edges of their instability strip as well as the periods range of their observed modes is explained by the balance between this driving mechanism and radiative damping in the g-mode cavity. Secondly, the multi-color photometric amplitude ratios and the phase differences between the light and velocity curves are considered. It is shown that the agreement between theory and observations obtained with TDC models is much better than with Frozen Convection (FC) models. The theoretical analysis of these observables makes the photometric identification of the degree  $\ell$  of the modes possible and gives constraints on the characteristics of the convective envelope of these stars. Finally, the attractive potential of  $\gamma$  Dor stars as targets for asteroseismology is considered.

**Key words.** Stars: oscillations – Convection – Stars: interiors – Stars: variables: general

## 1. Characteristics of the class

$\gamma$  Dor stars are a class of main sequence variable stars with a range in spectral types of A7-F5 ( $7400 \text{ K} \geq T_{\text{eff}} \geq 6900 \text{ K}$ ) and of luminosity class IV-V, or V ( $0.7 \leq \log(L/L_{\odot}) \leq 1.05$ ) (Kaye et al. 1999). Their variability cannot be explained by spot models or rotational modu-

lation (Balona et al. 1996), nor by binarity effects. It is now clearly admitted that it is due to pulsations in high-order gravity modes. About 54  $\gamma$  Dor stars are confirmed to date (Henry et al. 2005).

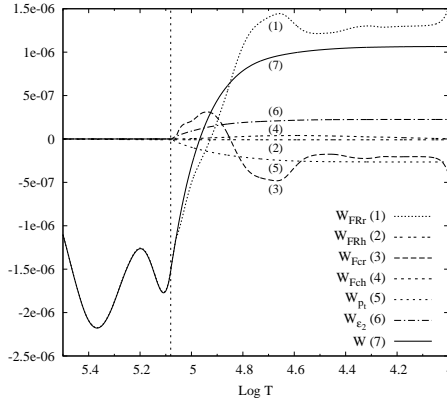
## 2. Driving mechanism

The driving mechanism of the  $\gamma$  Dor gravity modes has been a matter of debate during the

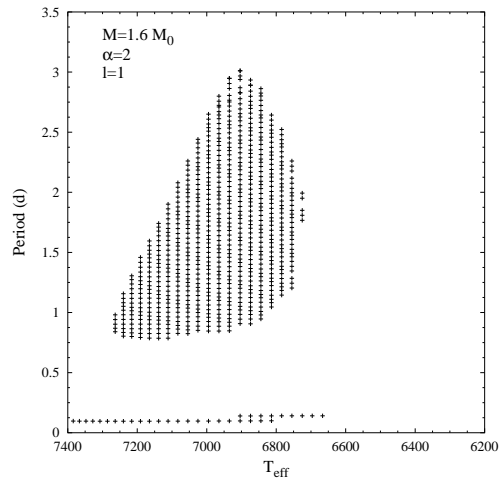
last decade. To understand it, we have to consider more closely the transition region where the pulsation periods are of the same order as the thermal relaxation time, because this region plays always the major role in the excitation or damping of the modes. The important point is that, for  $\gamma$  Dor stars, this transition region is near the bottom of the Convective Envelope (CE). This led Guzik et al. (2000, hereinafter G00) to explain as follows driving of the  $\gamma$  Dor g-modes. The radiative luminosity drops suddenly at the bottom of the convective zone. Therefore, at the hot phase of pulsation, the increasing energy coming from below the convection zone cannot be transported by radiation inside it. If we admit that the convective flux does not adapt immediately to the changes due to oscillations, the energy is thus periodically blocked and transformed in mechanical work leading to the oscillations.

G00 used a Frozen Convection (FC) treatment in their non-adiabatic modeling. In a very thin region near the bottom of the convection zone, the life-time of the convective elements is larger than the pulsation periods and the FC approximation is acceptable there; but it is the contrary in the rest of the convection zone. Hence, it is crucial to determine the exact role played by Time-Dependent Convection (TDC) in the driving mechanism of  $\gamma$  Dor stars. We have implemented in our linear non-radial non-adiabatic pulsation code the TDC treatment of Gabriel (1996) and Grigahcène et al. (2005). This treatment takes the time-variations of the convective flux ( $\delta F_c$ ), the turbulent pressure ( $\delta p_t$ ) and the dissipation rate of turbulent kinetic energy ( $\delta \epsilon_2$ ) into account.

In Fig. 1, we give the total work integral (curve 7) obtained with our TDC treatment for a typical  $\gamma$  Dor g-mode and a structure model with  $M = 1.6 M_\odot$ ,  $T_{\text{eff}} = 6935$  K,  $\log(L/L_\odot) = 0.96$  and  $\alpha = 2$  (model 1). Regions where the work integral increases (resp. decreases) have a driving (resp. damping) effect on the oscillations. We also give in this figure the contributions of the radial and transversal components of the radiative and convective flux perturbations. We see that significant driving occurs at the CE base for the mode  $\ell = 1$ ,  $g_{50}$  (Curve 7). Decomposition in radiative and con-



**Fig. 1.** Different physical components of the work integral obtained with our TDC treatment for the mode  $\ell = 1$ ,  $g_{50}$ , model 1. The vertical line is the CE base.



**Fig. 2.** Periods (in days) of the unstable gravity modes as a function of  $T_{\text{eff}}$  obtained for TDC models of  $1.6 M_\odot$  with  $\alpha = 2$ . Each cross corresponds to a given  $\ell = 1$  mode.

vective flux contributions shows that the convective flux variations do not play a significant role at the CE base (Curve 3). The main driving comes from the radiative flux variations (Curve 1) which supports the flux blocking mechanism proposed by G00. We see also in Fig. 1 that the transversal components of radiative and convective flux variations (Curves

2 and 4) do not play a significant role in the work integral, due to the fact that the horizontal wavelength ( $r/(\ell(\ell+1))$ ) is much larger than the scale heights of the different physical quantities in the superficial layers of the star. The contribution of  $\delta p_t$  and  $\delta \varepsilon_2$  on the work integral are given in Curves 5 and 6 respectively. We see that these terms have an opposite effect, so that the work integral that includes all the perturbed convection terms is close to the one with only the perturbed convective flux.

In Fig. 2, we show the periods range of the unstable modes predicted by our TDC models as a function of the effective temperatures for main sequence models of  $1.6 M_\odot$  with Mixing-Length (ML) parameter  $\alpha = 2$  and  $\ell = 1$  modes. The periods range and the effective temperatures for the unstable g-modes is in agreement with the typical observed periods in  $\gamma$  Dor stars for this value of  $\alpha$ . Theoretical instability strips for the  $\gamma$  Dor g-modes have been computed by Warner et al. (2003) using FC treatment and by Dupret et al. (2004, 2005a) using our TDC treatment. A good agreement with the observed instability strip can be obtained for  $\alpha \simeq 2$  (near the solar calibrated value). As shown by Dupret et al. (2004, 2005a), the theoretical instability strip is displaced towards lower effective temperatures when we decrease  $\alpha$ , simply because the size of the convective envelope (key point for the driving) is directly related to  $\alpha$ .

### 3. Damping mechanism

The stabilization of the high-order g-modes at the red side of the  $\gamma$  Dor instability strip and of the modes of intermediate radial order (modes with periods between 0.15 and 0.8 days in Fig. 2) is explained by a radiative damping mechanism occurring in the g-mode cavity. More precisely, the eigenfunctions spatial oscillations have large amplitudes in the g-mode cavity for these two cases. Hence, the radiative damping in this region is much larger than the excitation near the base of the convective envelope. We refer to Dupret et al. (2005a) for more details about this damping mechanism.

As a summary, the g-mode cavity always has a stabilizing influence, while the flux

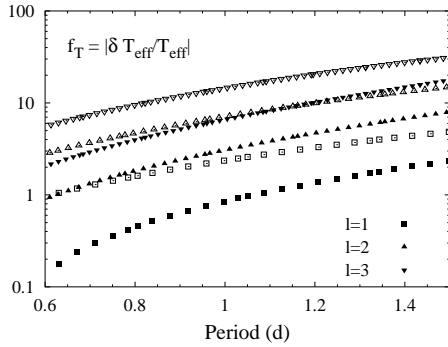
blocking at the bottom of the CE has a destabilizing one. The balance between these two mechanisms explains the exact location of the  $\gamma$  Dor instability strip.

### 4. Hybrid $\gamma$ Dor – $\delta$ Sct stars

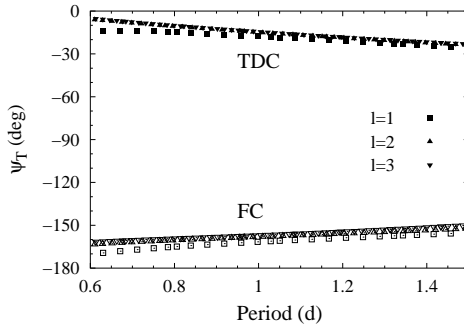
Our TDC models predict the existence of stars having simultaneously unstable high-order gravity modes of  $\gamma$  Dor type and unstable low-order p-g modes of  $\delta$  Sct type. This can be seen for example in Fig. 2, where the crosses at the bottom correspond to unstable  $\delta$  Sct type modes and the instability region between 0.8 and 3 days corresponds to  $\gamma$  Dor type modes. The detection of stars with such hybrid behaviour would present a very high interest for asteroseismology: their high order g-modes would enable us to probe the very deep layers of the star and their low-order p-g modes would enable us to probe the intermediate and superficial layers. Much observational effort has been performed to detect such hybrid stars and two have been discovered: HD 209295 (Handler et al. 2002) and HD 8801 (Henry & Fekel 2005). We refer to Grigahcène et al. (these proceedings) for more details about this aspect.

### 5. Mode identification

A crucial problem in asteroseismology is the mode identification. This problem is particularly difficult for  $\gamma$  Dor stars, because of the combined effect of rotation and convection on the frequencies, the amplitudes, the phases and the surface geometry of the modes. As shown by Mathias et al. (2004), many  $\gamma$  Dor stars show line-profile variations. Hence, spectroscopic mode identification can often be performed for these stars (Balona et al. 1996; Aerts & Krisciunas 1996; Jankov, these proceedings). On the other side, photometric mode identification methods are based on the analysis of the amplitude ratios and phase differences between different photometric passbands. An important point is that the latter observables are sensitive to the non-adiabatic treatment of convection-pulsation interaction.



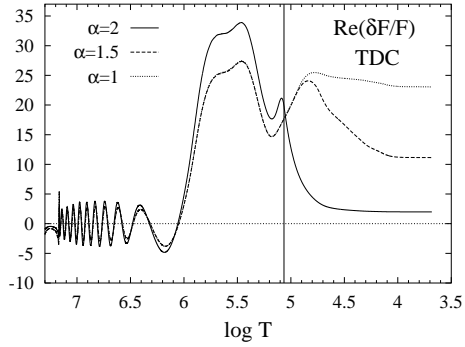
**Fig. 3.**  $f_T$  as a function of the period (in days), obtained with TDC treatment (full symbols) and FC treatment (empty symbols), model 2.



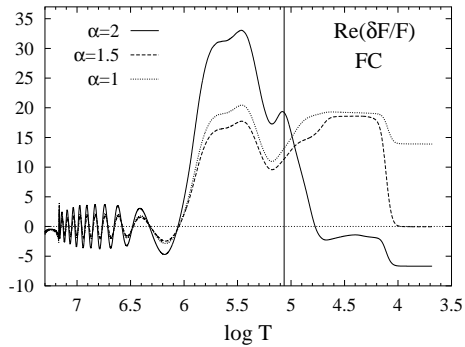
**Fig. 4.** Phase-lag  $\psi_T$  as a function of the period (in days), obtained with TDC treatment (full symbols) and FC treatment (empty symbols), model 2.

Hence, comparison with observations enables us to constrain this treatment.

Two important ingredients for the theoretical determination of the photometric amplitudes and phases are the normalized amplitude ( $f_T$ ) and phase ( $\psi_T$ ) of local effective temperature variation. These two quantities can only be obtained by non-adiabatic computations. We begin by giving in Figs. 3 and 4 the values of  $f_T$  and  $\psi_T$  as a function of the period in days, obtained with TDC and FC treatment, for modes of degree  $\ell = 1, 2, 3$  and a structure model with  $M = 1.55 M_\odot$ ,  $T_{\text{eff}} = 7020$  K,  $\log(L/L_\odot) = 0.87$ ,  $\alpha = 2$  and  $Z = 0.02$  (model 2). The values of  $f_T$  are smaller with TDC treatment than with FC treatment. As



**Fig. 5.**  $\Re\{\delta F/F\}$  (relative variation of the radial component of the total flux) as a function of  $\log T$ , obtained with TDC treatment, for the mode  $\ell = 1$ ,  $g_{22}$  ( $f = 1.192$  c/d) and models with different  $\alpha$ . The vertical line gives the bottom of the convective envelope, for the  $\alpha = 2$  model.



**Fig. 6.** Same caption as Fig. 5 but with FC treatment.

shown in Dupret et al. (2005b and these proceedings), these smaller values of  $f_T$  in the TDC case imply a better agreement with the typical observed photometric amplitude ratios of  $\gamma$  Dor stars. The values of  $\psi_T$  are completely different: they are close to  $0^\circ$  in the TDC case and close to  $180^\circ$  in the FC case. The phase-lags between light and velocity curves predicted by TDC models better agree with the typical observed phase-lags in  $\gamma$  Dor stars (Dupret et al. 2005b and these proceedings).

In Figs. 5 and 6, we give the real part of  $\delta F/F$  as a function of  $\log T$ , obtained with TDC and FC treatment. This eigenfunction is

normalized so that the relative radial displacement is 1 at the photosphere. Models are with  $T_{\text{eff}} = 7020$  K,  $\log(L/L_{\odot}) = 0.87$  and different values of the ML parameter  $\alpha$ . We recall that, at the photosphere,  $|\delta F/F| = 4 f_{\text{T}}$  and  $\phi(\delta F/F) = \psi_{\text{T}}$ . Comparing the results obtained for  $\alpha = 2$  (solid lines) enables us to explain the very different values of  $f_{\text{T}}$  and  $\psi_{\text{T}}$  obtained in the TDC and FC cases. In both cases, a first decrease of  $\Re(\delta F/F)$  occurs in the Fe partial ionization zone ( $\log T \approx 5.3$ ), it is due to a  $\kappa$ -mechanism similar to the case of Slowly Pulsating B stars. A second decrease of  $\Re(\delta F/F)$  occurs near the base of the Convective Envelope (CE) ( $\log T \approx 5$ ). This corresponds to the flux blocking mechanism. In the FC case,  $\kappa$ -mechanism occurs inside the convective envelope, in the partial ionization zones of HeII ( $\log T \approx 4.8$ ) and H ( $\log T \approx 4.1$ ). These  $\kappa$ -mechanisms imply additional decreases of  $\Re(\delta F/F)$  down to negative values, which explains the phase-lags around  $180^\circ$  predicted by the FC models with high  $\alpha$ . In contrast, these  $\kappa$ -mechanisms inside the CE are not allowed by TDC models, because they would lead to too high superadiabatic gradients. Therefore,  $\delta F/F$  remains flat and positive after the flux blocking drop and its phase remains near  $0^\circ$ . For smaller  $\alpha$ , the CE base is closer to the surface, so that the flux blocking is less efficient and the drop of  $\Re(\delta F/F)$  is smaller; hence, larger values of  $f_{\text{T}}$  are predicted and the change of sign of  $\Re(\delta F/F)$  no longer occurs in the FC case (for small  $\alpha$ ).

In Dupret et al. (2005b and these proceedings), the application to specific  $\gamma$  Dor stars is considered. It is shown that TDC results much better agree with observations, allowing a better identification of the degree  $\ell$  of the modes.

## 6. Effect of rotation

The long pulsation periods ( $\approx 1$  day) of the  $\gamma$  Dor stars are not much smaller than their rotation periods (a few days). Hence, the rotation-pulsation interaction is expected to be significant in these stars. There are two types of approaches for this interaction: perturbative and non-perturbative ones. The perturba-

tive approaches are based on series development in term of  $\sigma_{\text{rot}}/\sigma_{\text{pul}}$ . They have been derived up to the second order by Dziembowski & Goode (1992) and up to the third order by Soufi et al. (1998). Non-perturbative theories have been derived by Lee & Saio (1987) (LS) and by Dintrans & Rieutord (2000) (DR). The LS's theory is based on the use of the traditional, adiabatic and Cowling approximations. The DR's theory adopts the anelastic and adiabatic approximations; their geometric formalism is based on the integration of the characteristics propagating in the stellar interior. They applied their method to a typical  $\gamma$  Dor model and showed that the second order perturbative theory reaches its limits for rotation periods of about three days.

Many observables are strongly affected by rotation: the pulsation frequencies, the photometric amplitude ratios and phase differences, the line-profile variations, ... Following the LS's approach, Townsend (1997, 2003) determined the effect of rotation on these last observables which are widely used for mode identification. The main surface effect of rotation is to concentrate the oscillations along equatorial waveguides. This effect is expected to be significant in  $\gamma$  Dor stars and it would be important to take it into account in spectroscopic and photometric mode identification methods.

## 7. Asteroseismic potential

The high-order gravity modes of  $\gamma$  Dor stars have the highest inertia in the very deep layers near the top of the convective core. Hence, the frequencies of these modes give a unique opportunity to probe the physics of these layers in intermediate mass stars. In the asymptotic regime and if the effect of rotation is neglected, the frequencies of high-order gravity modes are approximately given by:  $\sigma_{n\ell} = \sqrt{\ell(\ell+1)}I/(\pi(n+1/2))$ , where  $I = \int_{r_a}^{r_b} N/r dr$  is the integral of the Brunt-Väisälä frequency  $N$  from the base to the top of the g-mode cavity (also referred to as the buoyancy radius). An original method based on this relation was recently proposed by Moya et al. (2005). For any couple of modes with same  $\ell$ , we have

$\sigma_{n_1}/\sigma_{n_2} = (n_2 + 1/2)/(n_1 + 1/2)$ . Different combinations of possible  $n$  can be determined by this way, and finally the constraints given by the buoyancy radius  $\mathcal{I}$  can be used to restrict the number of possible models for the star. The effect of rotation in the frame of this method was recently studied by Suárez et al. (2005). Finally, we note that a more complete seismic study can be performed by combining this method with a non-adiabatic analysis with TDC models. Such study makes it possible to obtain informations on both the deep interior and the treatment of convection in the superficial layers (Moya et al. these proceedings). Finally, we refer to Miglio et al. (these proceedings) who study the effects on gravity modes of the  $\mu$ -gradient region near the edge of the convective core. They show that the period and amplitude of the oscillatory component in the period spacing are directly related to the location and sharpness of the  $\mu$ -gradient.

## 8. Conclusions

The nature of the gravity modes pulsations in  $\gamma$  Dor stars begins to be better understood. TDC models confirm that their driving is due to a periodic flux blocking at the base of their convective envelope. The balance between this flux blocking driving and the radiative damping in the g-mode cavity explains the location of their instability strip. TDC models are required for the photometric mode identification in these stars. The rotation-oscillations interaction is expected to be significant and still much work has to be done at this level. Finally, we emphasize that these stars present a high potential for asteroseismology, as their g-modes give a unique opportunity to probe the deep layers near the convective core edge in young intermediate mass stars.

*Acknowledgements.* MAD acknowledges financial support from CNES. RG acknowledges financial support from the program ESP2001-4528-PE. JDR acknowledges support from the Fund for Scientific Research-Flanders.

## References

- Aerts, C., & Krisciunas, K. 1996, MNRAS, 278, 877
- Balona, L. A., Böhm, T., Foing, B. H., et al. 1996, MNRAS, 281, 1315
- Dintrans, B., & Rieutord, M. 2000, A&A, 354, 86
- Dupret, M.-A., Grigahcène, A., Garrido, R., et al. 2004, A&A, 414, L17
- Dupret, M.-A., Grigahcène, A., Garrido, R., et al. 2005a, A&A, 435, 927
- Dupret, M.-A., Grigahcène, A., Garrido, R., et al. 2005b, MNRAS, 360, 1143
- Dziembowski, W. A., & Goode, P. R. 1992, ApJ, 394, 670
- Gabriel, M. 1996, Bull. Astron. Soc. of India, 24, 233
- Grigahcène, A., Dupret, M.-A., Gabriel, M., et al. 2005, A&A, 434, 1055
- Guzik, J. A., Kaye, A. B., Bradley, P. A., et al. 2000, ApJ, 542, L57 (G00)
- Handler, G., Balona, L. A., Shobbrook, R. R., et al. 2002, MNRAS, 333, 262
- Henry, G. W., & Fekel, F. C. 2005, AJ, 129, 2026
- Henry, G. W., Fekel, F. C., & Henry, S. M. 2005, AJ, 129, 2815
- Kaye, A. B., Handler, G., Krisciunas, K., et al. 1999, PASP, 111, 840
- Lee, U., & Saio, H. 1987, MNRAS, 224, 513
- Mathias, P., Le Contel, J.-M., Chapellier, E., et al. 2004, A&A, 417, 189
- Moya, A., Suárez, J.-C., Amado, P. J., et al. 2005, A&A, 432, 189
- Soufi, F., Goupil, M.-J., & Dziembowski, W. A. 1998, A&A, 334, 911
- Suárez, J.-C., Moya, A., et al. 2005, A&A, accepted
- Townsend, R. H. D. 1997, MNRAS, 284, 839
- Townsend, R. H. D. 2003, MNRAS, 343, 125
- Warner, P. B., Kaye, A. B., & Guzik, J. A. 2003, ApJ, 593, 1049