Non-adiabatic seismic study of the thin convective envelope of δ Scuti stars

M.-A. Dupret, A. Grigahcène, R. Garrido

Instituto de Astrofísica de Andalucía-CSIC, Apartado 3004, 18080 Granada, Spain

J. Montalban, M. Gabriel, R. Scuflaire

Institut d'Astrophysique et de Géophysique, Université de Liège, allée du Six Août 17, B-4000 Liège, Belgium

Abstract. For δ Scuti stars, the theoretical predictions of a non-adiabatic pulsation code are very dependent on the characteristics of the thin convective envelope of the models (Balona & Evers 1999). The treatment of the non-adiabatic interaction between convection and pulsation has also a significant impact on the results, particularly near the red edge of the instability strip. The non-adiabatic theoretical predictions can be tested upon observations by comparing them to the amplitude ratios and phases differences as observed in different color passbands (Dupret et al. 2003). In the first part of this paper, we compare the results obtained by adopting different treatments of convection in the interior and atmosphere models: mixing-length theory (MLT) and full spectrum of turbulence (FST)(Canuto et al. 1996, CGM). In the second part of this paper, we examine the problem of the interaction between convection and pulsation and compare the modes stability obtained with and without including time-dependent convection in our non-adiabatic code.

1. Sensitivity of non-adiabatic pulsation observables on stationary convection models

1.1. Convection in interior and atmosphere stellar models

δ Scuti stars (A-F type stars) present a thin low efficient superficial convective zone extending up to the photosphere. In this paper, we compare the results obtained with different convection models in the interior and in the atmosphere of the star, following the MLT approach with $\alpha = l/H_p = 0.5$, $\alpha = 1.25$ or the FST approach. The interior models were computed by the evolutionary codes ATON3.0 (FST and MLT) and CLÉS (MLT). For the atmosphere models, we have used the ATLAS9 MLT models with $\alpha = 1.25$ (Kurucz 1993) and $\alpha = 0.5$ (Heiter et al. 2002); and the ATLAS9 FST models (CGM) (Heiter et al. 2002). All the models of this section are with $T_{\rm eff} = 7400$ and $\log g = 4.0$.
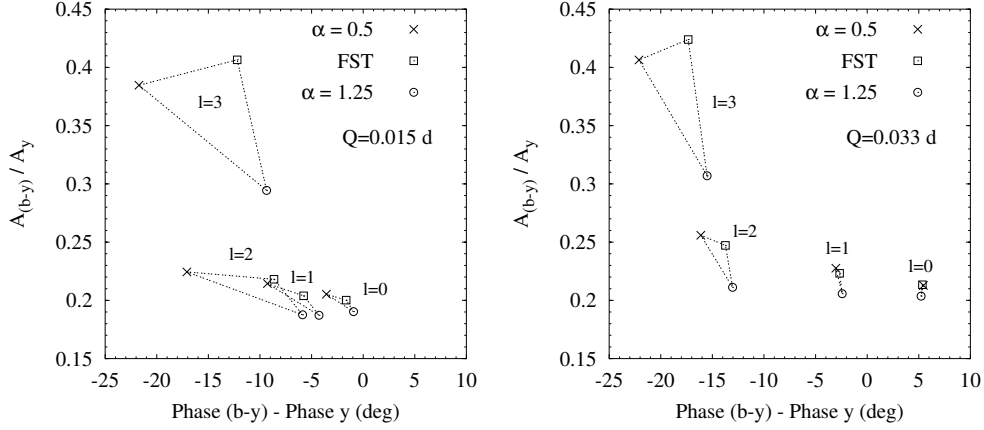
Figure 1.   Phase-amplitude diagrams obtained for different models of convection, for modes of degree $\ell = 0, 1, 2, 3$. The constant of pulsation $Q = 0.015$ days in the left panel and $Q = 0.033$ days in the right panel.

1.2.   Results

We performed non-adiabatic computations for all these models, using the non-adiabatic code MAD (Dupret et al. 2002). The phase-lags and the effective temperature variations predicted by the non-adiabatic pulsation code appear to be very sensitive to the details of the thin convective zone. Theoretical multi-color photometric amplitude ratios and phase differences depend on these non-adiabatic results and on the monochromatic flux and limb darkening of the atmosphere models (Dupret et al. 2003). The limb darkening coefficient for the FST models have been taken from Barban et al. (2003). In Fig. 1, we show the results obtained in phase-amplitude diagrams for Strömgren filters.

2.   Non-adiabatic interaction between convection and pulsation in δ Scuti stars

2.1.   Non-adiabatic convection theory of Gabriel (1996)

Turbulence is usually described in term of mean and fluctuation equations. In the theory of Gabriel (1996), the fluctuation equations of continuity, of motion and of energy conservation have respectively the following linearized form.

$$\nabla \cdot \vec{V} = 0 \,, \tag{1}$$

$$\overline{\rho} \frac{d\vec{V}}{dt} = \frac{\Delta \rho}{\rho} \nabla \overline{P} - \nabla \Delta P - \frac{8\rho \vec{V}}{3\tau_{\mathrm{c}}} - \rho \vec{V} \cdot \nabla \vec{u} \,, \tag{2}$$

$$\frac{\Delta\left(\rho T\right)}{\overline{\rho T}} \frac{d\overline{s}}{dt} + \frac{d\Delta s}{dt} + \vec{V} \cdot \nabla \overline{s} = -\frac{\Gamma^{-1} + 1}{\tau_{\mathrm{c}}} \Delta s \,, \tag{3}$$

where τ_{c} is the life time of the convective elements and Γ is the convective efficiency.

Stationary solutions of the above equations lead to the classical MLT theory of Böhm-Vitense. The lagrangian perturbation of the mean equations gives the linear pulsation equations, where appear unknown terms such as the perturbation of the convective flux:

$$\delta \vec{F}_{\rm c} = \overline{\vec{F}}_{\rm c} \left(\frac{\delta c_p}{c_p} + \frac{\delta \rho}{\overline{\rho}} + \frac{\delta T}{\overline{T}} \right) + c_p \overline{\rho} \, \overline{T} \left(\overline{\delta \Delta S \vec{V}} + \overline{\Delta S \delta \vec{V}} \right) . \qquad (4)$$

The unknown correlation terms of Eq. (4) can be obtained from the fluctuation equations. More precisely, we perturb Eqs. (1) to (3) and search for solutions of the form $\delta \left(\Delta X \right) = \delta \left(\Delta X \right)_{\vec{k}} \, e^{i \, \vec{k} \cdot \vec{r}} e^{i \sigma \, t}$. Then we integrate these particular solutions over all values of k_θ and k_ϕ such that $k_\theta^2 + k_\phi^2 = A \, k_r^2$ ($A = 1/2$ for an isotropic turbulence). Finally, the perturbed convective flux takes the following form:

$$\delta \vec{F}_{\rm c} = \delta F_{\rm cr} \left(r \right) Y_l^m \left(\theta, \phi \right) \, \vec{e}_r + \delta F_{\rm ch} \left(r \right) \left(r \, \nabla_h Y_l^m \left(\theta, \phi \right) \right)$$

and the problem is naturally separated in spherical harmonics. $\delta F_{\rm cr} \left(r \right)$ and $\delta F_{\rm ch} \left(r \right)$ are related to the perturbed mean quantities by first order differential equations.

The main source of uncertainty of any MLT theory of convection-pulsation interaction is in the way to perturb the mixing-length l. Gough (1977) proposes to perturb only the pressure scale height: $\delta l / l = \delta H_{\rm p} / H_{\rm p}$. On the other hand, it can be expected that the perturbation of the mixing-length becomes negligible when the life-time of the convective elements is much longer than the period of pulsation, this can be reproduced for example by adopting: $\delta l / l = 1 / \left(1 + \left(\sigma \tau_{\rm c} \right)^2 \right) \delta H_{\rm p} / H_{\rm p}$.

2.2.   Red edge of the δ Scuti instability strip

A very good confirmation of the quality of our time-dependent convection treatment is that it is able to reproduce the red edge of the δ Scuti instability strip, contrary to models with frozen convection. In Fig. 2, we give the evolution of the dimensionless frequencies ($\omega = \sigma \, t_{\rm dyn}$) of $\ell = 2$ modes as a function of effective temperature and age, for $1.8 \, M_\odot$ models with overshooting $\alpha_{\rm ov} = 0.2$ and mixing-length parameter $\alpha = 1.5$. The right (resp. left) panel corresponds to the results obtained with (resp. without) including time-dependent convection in our non-adiabatic code.

3.   Conclusions

On one hand, the theoretical non-adiabatic amplitudes and phases of effective temperature variations, as well as the monochromatic flux and limb darkening derivatives are very sensitive to the stationary convection models, in the thin convective envelope of δ Scuti stars. This has a significant impact on the photometric amplitude ratios and phase differences as observed in different color passbands. A confrontation to accurate observations could thus allow to discriminate between these different convection models.

On the other hand, our non-adiabatic time-dependent treatment of the interaction between convection and pulsation allows us to predict a red edge of
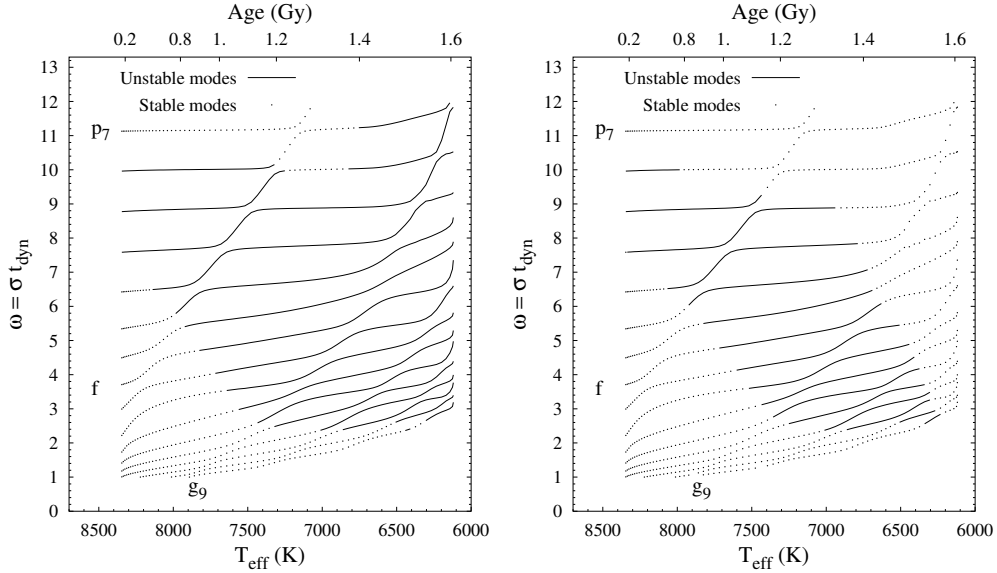
Figure 2.   Stable and unstable modes obtained with (right) and without (left) including time-dependent convection in the models.

the δ Scuti instability strip in agreement with observations, for radial as well as for non-radial modes. This red edge cannot be obtained with frozen convection models.

References

Balona, L. A., & Evers, E. A. 1999, MNRAS, 302, 349

Barban, C., Goupil, M. J., Van't Veer-Menneret, C., Garrido, R., Kupka, F., & Heiter, U. 2003, A&A, 405, 1095

Canuto, V. M., Goldman, I., & Mazzitelli, I. 1996, ApJ, 473, 550

Dupret, M.-A., De Ridder, J., Neuforge, C., Aerts, C., & Scuflaire, R. 2002, A&A, 385, 563

Dupret, M.-A., De Ridder, J., De Cat, P., Aerts, C., Scuflaire, R., Noels, A., & Thoul, A. 2003, A&A, 398, 677

Gabriel, M. 1996, Bull. Astron. Soc. Of India, 24, 223

Gough, D. O. 1977, ApJ, 214, 196

Heiter, U., Kupka, F., van't Veer-Menneret, C., Barban, C., et al. 2002, A&A, 392, 619

Kurucz, R. L. 1993, ATLAS9 Stellar Atmosphere programs and 2 km/s grids. Kurucz CDROM No 13