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Influence of the equation of state on the solar five-minute oscillation

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Summary. Solar models have been computed with an equation of state taking the electrostatic corrections into account for Z equal to 0.018 and 0.02. They lead to initial hydrogen abundances lower than found by Shibahashi et al. (1983) and by Ulrich and Rhodes (1983). The predicted frequencies in the five-minute range are too small by 5–10 μHz for $l \leqq 3$ and by 10–20 μHz for $l=10$ and 20. For high values of l, the frequencies are too large by a few percent. Our results have been compared with those of other groups and the influence of the numerical code used for the computation of the frequencies is discussed. It is shown that differences between codes are not responsible for differences in the frequencies larger than 2 μHz. Larger deviations must find their origin in physical differences between models. The deviations between theoretical frequencies are however smaller than the discrepancy with observation and all standard solar models fail to reproduce the observations. It is therefore concluded that something is missing in the physics of solar evolution. This missing effect should lead to a model with a deeper convective zone.

Key words: stellar structure – Sun – five-minute solar oscillation – stellar stability

1. Introduction

Since their discovery in 1979 (Claverie et al., 1979), the low l, 5min solar oscillations have been extensively observed by several groups (see Claverie et al., 1979, 1980, 1981; Grec et al., 1980, 1982; Scherrer et al., 1982; Woodard et al., 1982; Duvall and Harvey, 1983). The comparison of the observed frequencies show that an accuracy of at least 10^{-3} (i.e. better than 3 μHz) is achieved. For the first time, a large number of very accurate data provide constraints for the internal structure of the Sun.

The first comparisons of these observations with the theoretical predictions (see Christensen-Dalsgaard and Gough, 1980, 1981; Ulrich and Rhodes, 1982, 1983; Shibahashi and Osaki, 1981; Shibahashi et al., 1983; Scuflaire et al., 1981, 1982; Gabriel et al., 1982) show a fairly good agreement, better than 1 %, but also a significant discrepancy. We first thought that improvements in the physics could reduce the small gap between theory and observations. Models computed by Shibahashi et al. (1983) taking the electrostatic corrections into account and using the Planck-Larkin partition functions (see Ulrich, 1982) did provide a very significant improvement. As the correct expressions for the partition functions is questionable, we could hope that computations of models with other partition functions would provide significantly different results. We therefore resumed the problem with partition functions based on Debye shielding.

Such models were computed for $Z=0.018$ and 0.02 (see Sect. 2). The following sections present the frequency calculations for low, intermediate and high l together with the discussion of the accuracy tests.

2. Equilibrium models

The evolutions have been computed with the Henyey code (Henyey et al., 1964, 1965). An important modification has however been introduced. The structure of models with convective zones is no longer found by interpolations between two or three models having convective zones boundaries bracketting the real ones. The position of convective zone limits are now moved by interpolation at each iteration and a double mesh point is placed at each limit. Therefore, the correct model is immediately obtained at the end of the relaxation process. The atmosphere is still computed according to Henyey et al. (1965).

The physics has been improved by the introduction of the electrostatic corrections in the equation of state and of internal partition functions for H and He which are no longer given by the statistical weight of the fundamental level. Starting from an expression of the free energy, all the properties of the gas are computed in a thermodynamically coherent way (Gabriel, 1967a, b, 1968; Graboske et al., 1969; Fontaine et al., 1977; Däppen, 1980). Our expression of the free energy F neglects He effects for excluded volume and is given by

$$F=F_{pg}-\frac{V}{3}\frac{e^2}{R_D}\left(\sum_i n_i Z_i^2+n_e\right), \tag{1}$$

where F_{pg} is the free energy of the perfect gas mixture. The second term gives the electrostatic correction according to the Debye-Hückel theory. V is the volume of the gas, e the electron charge, n_e the number density of electrons, n_i the number density of ions with charge $Z_i e$ and R_D the Debye radius given by

$$R_D=\left[\frac{4\pi e^2}{kT}\left(\sum_i n_i Z_i^2+n_e\right)\right]^{-1/2}. \tag{2}$$

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Table 1. Main properties of the solar models

Z	X	ϱ_c	$T_c(10^6)$	X_c	d_e (km)	q_e	ϱ_e	$T_e(10^6)$	L	R
0.018	0.708	150.2	15.63	0.3448	1.72 10^5	0.0136	0.121	1.838	3.86	6.95
0.020	0.698	151.5	15.75	0.3298	1.75 10^5	0.0151	0.125	1.917	3.86	6.93

Table 2. Frequencies of low and intermediate l modes for $Z=0.018$

l	$l=0$	$l=1$	$l=2$	$l=3$	$l=4$	$l=5$	$l=10$	$l=20$
9					1631.1	1675.5		
10			1665.7	1720.1	1768.1	1812.4		
11	1678.1	1740.9	1800.9	1854.8	1903.7	1948.9	2147.0	2480.1
12	1812.8	1875.1	1935.4	1990.2	2040.0	2086.0	2287.1	2624.8
13	1947.2	2010.2	2070.8	2126.1	2176.6	2223.2	2426.0	2769.9
14	2082.4	2145.7	2206.5	2261.9	2312.3	2359.1	2563.6	2916.0
15	2217.7	2280.7	2341.1	2396.7	2447.5	2494.6	2701.5	3061.3
16	2352.0	2415.2	2475.3	2530.7	2581.7	2629.3	2840.8	3206.1
17	2485.7	2548.5	2608.7	2664.6	2716.4	2765.1	2980.8	3350.7
18	2618.9	2682.0	2743.0	2799.8	2852.7	2902.4	3120.3	3495.3
19	2752.9	2816.9	2878.9	2936.4	2989.6	3039.5	3259.8	3640.2
20	2888.7	2953.0	3014.8	3072.7	3126.3	3176.6	3399.4	3785.1
21	3024.3	3088.9	3150.7	3208.9	3263.1	3313.9	3539.5	3929.7
22	3159.8	3224.7	3286.9	3345.4	3400.0	3451.1	3680.1	4074.1
23	3295.7	3360.8	3423.1	3482.4	3537.5	3589.6	3820.9	
24	3431.6	3497.3	3560.1	3619.6	3675.5	3728.2		
25	3568.5	3634.3	3697.5	3757.5	3813.7	3866.8		
26	3705.6	3771.8	3835.2	3895.2	3951.9	4005.3		
27	3842.9	3909.1	3972.7	4033.1	4090.2	4144.1		
28	3980.2	4046.6	4110.5	4171.1	4228.3	4282.7		
29	4117.8	4184.1	4248.1	4309.1	4366.9	4421.6		
30	4255.2	4321.9	4386.2	4447.4	4505.4	4560.3		
31	4393.0	4459.8	4524.1	4585.3	4642.9	4696.0		
32	4530.7	4597.2	4660.7	4717.8	4791.8	4843.5		
33	4666.9	4727.0	4807.5	4866.7	4924.8	4980.3		
34	4813.3	4877.9	4941.9	5003.5	5062.2	5118.2		
35	4947.9	5014.4	5078.9	5140.7				
36	5084.8	5151.4						

From Eq. (1), the properties of the gas are given by

$$\frac{E}{T^2} = -\left(\frac{\partial \frac{F}{T}}{\partial T}\right)_{V,N_i}, \quad (3)$$

$$S = -\left(\frac{\partial F}{\partial T}\right)_{V,N_i}, \quad (4)$$

$$p = -\left(\frac{\partial F}{\partial V}\right)_{T,N_i}. \quad (5)$$

Each modified Saha equation is given by

$$\sum_i \nu_i \left(\frac{\partial F}{\partial N_i}\right)_{T,V,(N)} = 0, \quad (6)$$

where ν_i is the stoechiometric coefficient of element i in the ionisation reaction considered.

The internal partition function of atoms i are given by

$$B_i = \sum_j g_j^i e^{-\frac{\Delta\chi_j^i}{kT}}, \quad (7)$$

where $\Delta\chi_j$ is the excitation potential of level j.

The summation is limited to levels verifying the condition

$$\chi_i - \frac{e^2 Z_i}{R_D} > \Delta\chi_j. \quad (8)$$

For He II, an effective Z has been used in Eq. (8) such that its ionisation potential is equal to that of a hydrogenic atom of charge Z_i.

With those definitions, B_i are functions of T and R_D, continuous in T at constant R_D. However due to condition (8), they are step functions of R_D at constant T. This unphysical property has been extensively discussed by Däppen (1980). To remove it, we have

Table 3. Frequencies of low and intermediate l modes for $Z=0.02$

l	$l=0$	$l=1$	$l=2$	$l=3$	$l=4$	$l=5$	$l=10$	$l=20$
9					1632.3	1677.0		
10			1666.9	1721.6	1769.8	1813.9		
11	1679.3	1742.2	1801.9	1856.1	1905.4	1905.6	2149.3	2482.3
12	1813.8	1876.4	1936.8	1991.4	2041.1	2087.1	2289.2	2627.0
13	1948.4	2011.1	2071.8	2127.5	2178.2	2225.0	2428.1	2771.9
14	2083.3	2146.9	2208.0	2263.3	2313.6	2360.6	2565.6	2917.7
15	2219.1	2281.8	2342.2	2398.1	2449.0	2496.0	2703.4	3068.5
16	2353.1	2416.3	2476.4	2531.9	2583.2	2630.8	2842.4	3207.8
17	2486.8	2549.6	2609.9	2665.8	2717.7	2766.4	2982.5	3352.5
18	2620.0	2683.0	2744.0	2800.6	2853.4	2903.1	3121.8	3497.0
19	2753.9	2817.5	2879.2	2937.0	2990.4	3040.6	3261.0	3641.8
20	2888.9	2953.5	3015.5	3073.5	3126.9	3177.1	3400.2	3786.5
21	3024.9	3089.4	3151.0	3209.1	3263.5	3314.4	3540.4	3931.0
22	3160.0	3224.8	3287.1	3345.5	3400.1	3451.3	3680.6	4075.0
23	3295.8	3360.6	3423.0	3482.3	3537.5	3589.8	3821.5	
24	3431.4	3497.1	3560.0	3619.5	3675.1	3727.8		
25	3568.2	3633.9	3696.9	3757.0	3813.6	3866.6		
26	3704.8	3771.2	3834.7	3894.8	3951.4	4004.8		
27	3842.4	3908.5	3972.0	4032.3	4089.5	4143.6		
28	3979.4	4045.6	4109.7	4170.4	4227.6	4282.0		
29	4116.9	4183.3	4247.2	4308.2	4365.9	4420.8		
30	4254.1	4320.7	4385.0	4446.4	4504.4	4559.2		
31	4391.7	4458.6	4522.8	4584.0	4641.9	4695.2		
32	4529.3	4595.8	4659.5	4716.4	4789.6	4841.7		
33	4665.7	4725.2	4805.3	4864.8	4923.2	4979.2		
34	4811.0	4875.8	4940.2	5002.2	5061.0	5117.0		
35	4946.2	5013.0	5077.5	5139.4				
36	5083.3	5149.8						

used for B_i the continuous function defined by the lowest value of B_i at each discontinuity. This assumption is of course not really satisfactory but the theory contains more important weaknesses.

Condition (8) which results from the solution of a hydrogenic atom in a Debye potential, predicts the disappearance of the fundamental level when the Debye radius is smaller than a_0/Z_i where a_0 is the Bohr radius.

When $R_D=a_0/S_i$, the sphere of radius a_0/Z_i contains several particules which means that the influence of the electrostatic interactions in the energy level are probably not well represented by a stable potential.

Ulrich (1982) has introduced in astrophyscis the Planck-Larkin's partition functions, also used in Shibahashi et al. (1983), which are function of temperature only. Such a choice has been criticized by Rouse (1983). These remarks show that a better theory of the internal partition functions is needed.

With this expression of B_i, ionization of H and He does not increase continuously inward. B_i defined in Eq. (7) has therefore been multiplied by $\exp\left[-\alpha\left(\frac{a_0}{\bar{a}}\right)^3\right]$ where $\bar{a}$ is the mean distance between ions and α is taken equal to 1.5.

From (1) and (5) p is given by

$$p=p_{pg}-\frac{1}{6}\frac{e^2}{R_D}\left(\sum_i n_i Z_i^2+n_e\right)-\sum_i n_i kT\left(\frac{\partial \ln B_i}{\partial \ln \varrho}\right)_{T,N_i}. \tag{9}$$

The two extra terms in this equation contribute respectively at most to 5.5% and 2% of the total pressure. This maximum is reached at $T\simeq 5\ 10^4$ K. The first of these terms gives a negative contribution to the pressure while the second one increases p as

$$\left(\frac{\partial \ln B_i}{\partial \ln \varrho}\right)_{T,N_i}<0.$$

The opacity coefficients are interpolated in tables computed from the Los Alamos Opacity Library (Heubner et al., 1977). In a first step, $\log \kappa$ is interpolated linearly in terms of $\log \varrho$ and $\ln(\varrho/T^3)$. κ is then interpolated linearly in term of X between 3 tables computed for $X=0.8$, 0.5, and 0. At temperatures below 10^4 K, Alexander's (1975) tables are used.

The nuclear reaction rates are taken from Fowler et al. (1975).

The solar models corresponding to $Z=0.018$ and 0.02 have an age of $4.62\ 10^9$ yr. L and R are adjusted according to the relations

$$\begin{aligned}\ln L_\odot&=\ln L-8.76(X-X_\odot)+1.6175\ 10^{-2}(l-l_\odot)\\ \ln R_\odot&=\ln R-1.7275(X-X_\odot)-1.1080\ 10^{-1}(l-l_\odot)\end{aligned} \tag{10}$$

whose coefficients come from two trial evolutions.

Table 1 gives the main properties of the models. The first two columns give the initial composition, the next three the central density, temperature and hydrogen abundance, the next four give the depth d_e of the convective envelope, its mass fraction, the density and the temperature at its base. Our model for $Z=0.018$ is very close to that found by Bahcall et al. (1982). It has a central density 4% lower and a central temperature higher by $9\ 10^{-3}$. The discrepancy remains of a few per cent throughout the model except

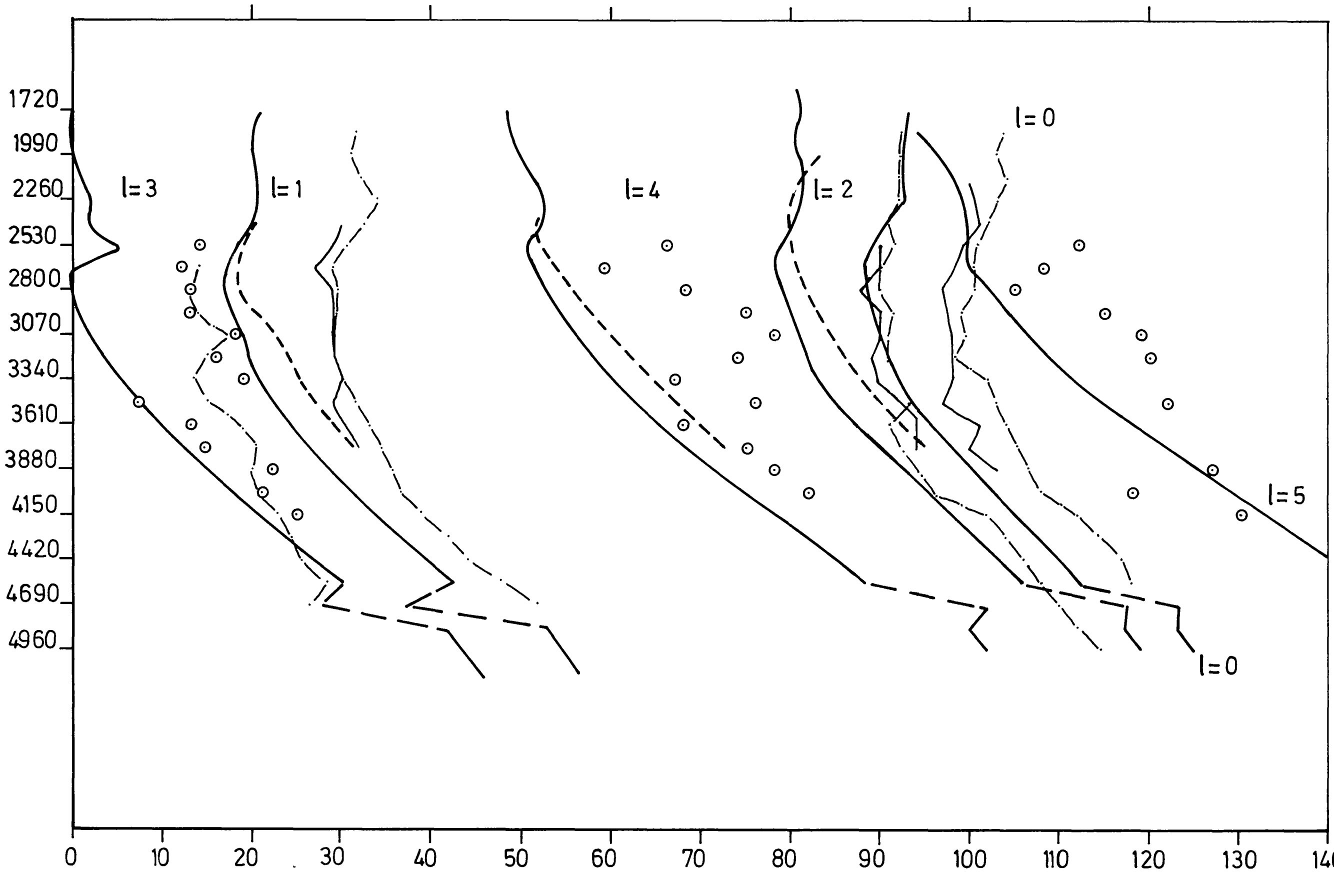

Fig. 1. Folded spectrum for the model with $Z=0.018$. The heavy full lines represent our theoretical results; the thin full lines, the observations by Claverie et al. (1981), the dot-dashed curves, the observations by Grec et al. (1982). Those of Sherrer et al. (1982) are given by open circles. The broken lines are Ulrich and Rhodes' (1983) results

maybe in the convective envelope where the comparison could not be made.

Our X value is however lower by 0.024. When compared with Shibahashi et al.'s (1983), our models have X value smaller by 0.045 and central density and temperature lower by 25% and 3%, respectively. The cause of these differences is difficult to trace out.

As Ulrich and Rhodes (1983) and Shibahashi et al. (1983) use the Planck-Larkin partition function instead of our relation (7), we first checked the influence of the partition function. A solar model was evolved with $Z=0.018$, $X=0.708$ and gives at the solar age $L/L_\odot=1.008$, $R/R_\odot=0.990$. According to Eq. (11) an increase of X by 7.2 10^{-4} will give the solar luminosity and radius.

To test the accuracy of our Henyey code, we have recomputed the solar model using a fitting procedure with the surface conditions replaced by the atmospheres square of the Henyey method and the $X(m)$ profile furnished by the evolution. The solution reproduces the solar model which was relaxed to 10^{-5} to better than 10^{-3}.

For the oscillation calculations, a chromosphere whose temperature profile $T(r)$ was taken from Vernazza et al. (1973), was added to the model.

3. The low l spectrum

The frequencies have been computed as in our previous papers. They are given in Tables 2 and 3. The folded spectrum comparison is shown in Figs. 1 and 2. The heavy lines represent our results, the thin full lines the observations by Claverie et al. (1981), the dot dashed curves Grec et al.'s (1982) observations. These of Scherrer et al. (1982) are given by circles. Figure 1 ($Z=0.018$) reproduces Ulrich and Rhodes' (1983) results (broken lines) for $l=1$, 2, and 4. In Fig. 2 ($Z=0.02$), we give also our previous results (Scuflaire et al., 1981) for $l=1$, 2, and 3 (broken lines). It can be seen that our frequencies are systematically too small while the discrepancy is much larger than the observational errors. The shape of the curves however are now more similar to the observed one. Nevertheless the difference in the curvatures remains large enough to prevent the possibility of realizing the agreement by a small change in the solar frequencies scale factor $(GM/R^3)^{1/2}$. The deviation of our results from these of Ulrich and Rhodes (1983) can reach about 5 μHz but more important is the difference in the shapes of the curves. The comparison with Shibahashi et al. (1983) shows that the shape of the folded spectrum are rather similar but that our frequencies are smaller by about 4 μHz for $l=0$ with a difference increasing to about 12 μHz for $l=4$. The sensitivity of the frequencies to Z is found to be very small as they differ by less than 1 μHz for the 2 values considered here. In Shibahashi et al. (1983) the variation was of several μHz.

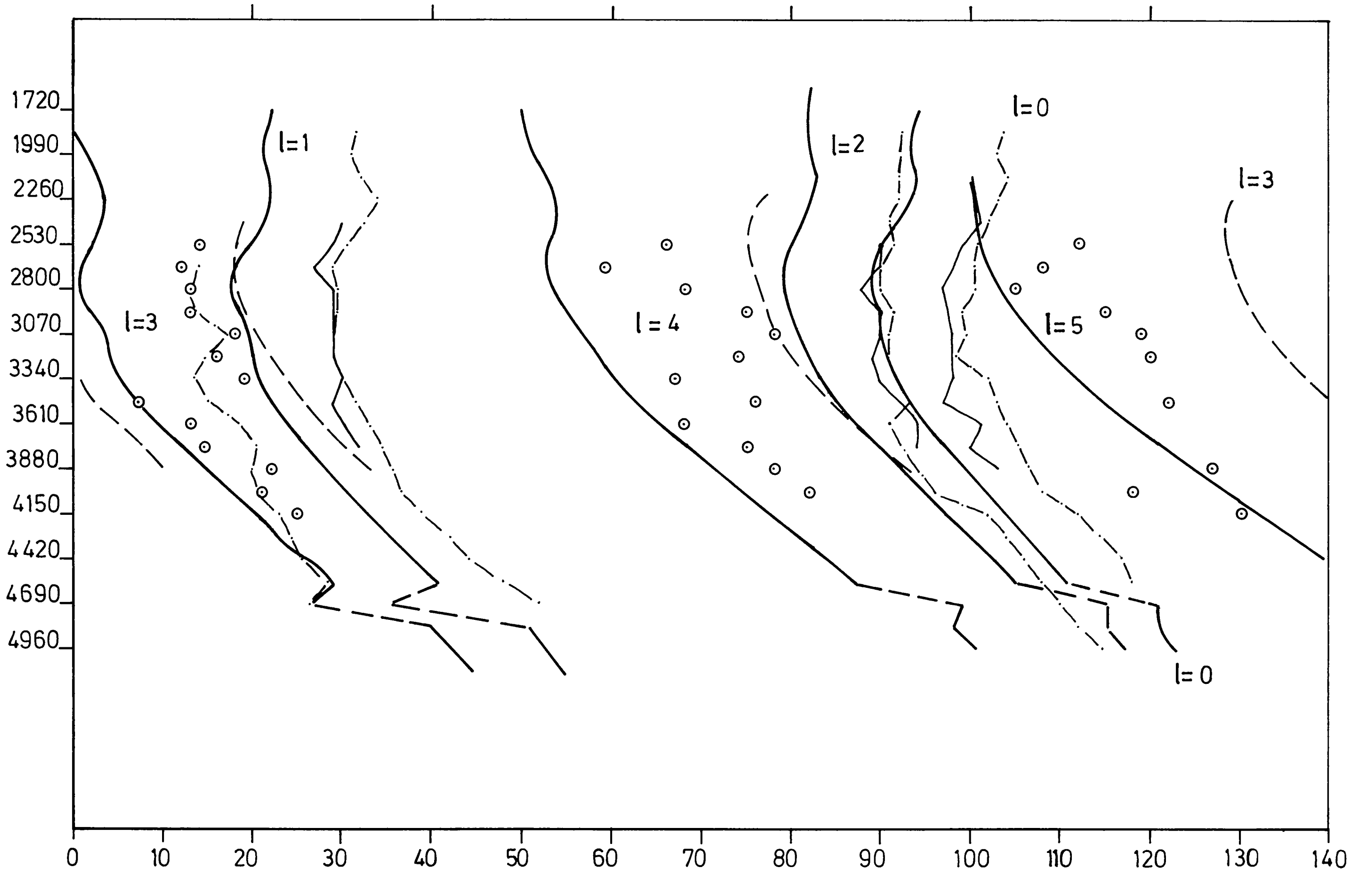

Fig. 2. Same as Fig. 1 for the model with $Z=0.02$, except that the broken lines give Scuflaire et al.'s (1981) results

Table 4. Comparison between Shibahashi and Osaki's code and ours

l	n		$\Delta\nu$(μHz)	600–1200	$\Delta\nu$(μHz)	900–1200	$\Delta\nu$(μHz)
2	10	1664.1	−1.7	1664.3	−1.4	1664.3	−1.4
	15	2339.3	−1.8	2339.9	−1.2	2339.7	−1.4
	20	3013.3	−1.5	3014.6	−0.2	3014.0	−0.8
	25	3359.7	−0.4	3562.2	+2.1	3560.9	+0.8
4	19	2990.6	+1.0	2889.1	−0.5	2988.5	−1.1
	20	3127.6	+1.3	3126.2	−0.1	3125.5	−0.8
	21	3265.2	+2.1	3264.0	+0.9	3263.1	0.0
	22	3401.4	+1.4	3401.8	+1.8	3400.8	+0.8

To check whether these differences between theoretical results are due to differences between the models or to our stability program, several tests have been made.

Firstly, different surface boundary conditions were used and applied at different points in the model. The results are shown in Fig. 3 for the $l=2$, $n=27$ mode. It gives the difference $\delta\nu$ between the frequency obtained when the boundary condition is applied at a level $z=(r-R)/R$ and the value given in Table 2. The surface conditions are respectively the isothermal condition (I), the regularity condition for zero surface temperature models (Z), the annulation of the radial (DR) and of the tangential displacement (DT). It shows that, when applied in the chromosphere, the influence of the boundary conditions is smaller than 1 μHz. This confirms the results of Ulrich and Rhodes (1983).

Secondly, the program was tested using the homogeneous model. In that case, the frequencies are found to better than 10^{-5} for $n \simeq 25$ for models containing 300 points, provided that most of the points are in the outer 10% of the radius. This condition is fulfilled by our solar model as on a total of 300 points, 230 are in the convective envelope and 210 have $r/R>0.9$. Moreover the stability program adds points by a cubic interpolation formula in order to have enough points between 2 successive nodes.

Thirdly, Shibahashi and Osaki's program was used to computed the frequencies of the model with $Z=0.018$. The results are

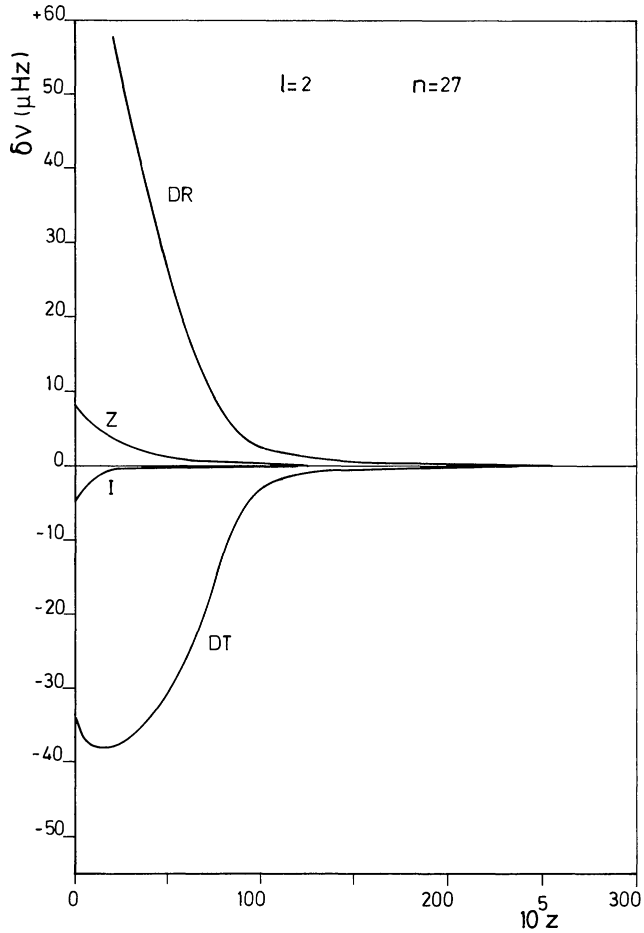

Fig. 3. Variation of the eigenfrequency in terms of the point where the surface boundary condition is applied. The surface conditions are respectively the isothermal condition (I), the regularity condition for zero surface temperature models (Z), the annulation of the radial (DR) and of the tangential displacement (DT)

shown in Table 4. The first set of results were computed after adding points in each shell such that

$$0.1 < \left[\left|(\sigma^2 - N^2)\left(\frac{l(l+1)}{r^2\sigma^2} - \frac{1}{c^2}\right)\right|\right]^{1/2} (r_{J+1} - r_J) < 0.3.$$

The second and third sets of results are obtained by extrapolating σ^2 for an infinite number of mesh points (see Shibahashi and Osaki, 1981), from models with 600 and 1200 points and with 900 and 1200 points respectively. $\Delta\nu$ gives the deviation from our results. It is seen that $\Delta\nu$ does not exceed 2 μHz.

We can therefore conclude that most of the differences between the frequencies found by different groups result from differences in their models.

From the data of Tables 2 and 3, it is possible to compute the mean frequency separation between 2 successive modes of given l, the average distance between modes of $l=1$ and 0, $\overline{\Delta\nu_{10}}$, or $l=2$ and 1, $\overline{\Delta\nu_{21}}$, at given n, or between modes of $l=0$ and the closest $l=2$ frequency, $\overline{\Delta\nu_{02}}$.

The averages are taken in the range $\nu = 2.4$–3.85 mHz for comparison with the data of Claverie et al. (1981). $\overline{\Delta\nu_{10}}$, $\overline{\Delta\nu_{21}}$, $\overline{\Delta\nu_{02}}$ defined as in Shibahashi et al. (1983) are given in Table 5. The frequency separations at constant l are too large by about 0.4 μHz, $\overline{\Delta\nu_{10}}$ is too small, $\overline{\Delta\nu_{21}}$ is within the observational error.

Table 5. Mean frequency separations

	Observations	$Z=0.018$	$Z=0.02$
$l=0$	135.2 ± 0.2	135.7	135.6
$l=1$	135.2 ± 0.1	135.6	135.5
$l=2$	135.4 ± 0.2	136.0	135.8
$\overline{\Delta\nu_{10}}$	65.3 ± 0.4	64.6	64.3
$\overline{\Delta\nu_{21}}$	61.6 ± 0.4	61.9	61.9
$\overline{\Delta\nu_{02}}$	8.3 ± 0.3	9.4	9.3

As from the definition of the $\overline{\Delta\nu_{ij}}$, we must have

$$\overline{\Delta\nu_{02}} \simeq \overline{\Delta\nu_{21}} - \overline{\Delta\nu_{10}}, \quad \overline{\Delta\nu_{02}}$$

is expected to be too large, as verified. These values are very close to those of Shibahashi et al. (1983) except for $\overline{\Delta\nu_{21}}$ which is much closer to the observed value.

In addition to the frequencies given in Tables 2–4, two additional pseudo chromospheric modes were found. One for $3467 < \nu < 3468$ μHz, the other for $4748 < \nu < 4764$ μHz.

They have no physical meaning as they are present in our calculation only because we use a zero temperature boundary condition at the outermost point of the model. The first one has no influence on the behaviour of the curves in Figs. 1 and 2.

The second one perturbs the closest eigenvalue and produce the zigzag in the curves for $4690 < \nu < 4825$ μHz.

4. Intermediate *l* oscillations

Recently Duvall and Harvey (1983) have observed oscillations in the whole range of l lower than 100.

Tables 2 and 3 give the frequencies predicted by our models for $l=10$ and 20. Figure 4 gives the comparison with the observations and the theoretical results of Ulrich and Rhodes (1983). Again our frequencies become more and more smaller than these of Ulrich and Rhodes as the number of radial nodes n increases. In both sets of theoretical frequencies, the discrepancy is larger than for low l and it is therefore impossible to reproduce the observations.

5. High *l* oscillations

The high l modes were computed as described in Gabriel and Noels (1976) but using the variables of Gabriel and Scuflaire (1979). Results are given in Tables 6 and 7. Comparison with results obtained by a completely different program, using a RKG integration scheme, show that they are accurate to better than 10^{-3}. Compared with Shibahashi et al. (1983), a good agreement is found for low frequencies but our values become up to 2 % larger for high l and high frequencies. When compared with the observations, an agreement at the 1 % level is found for frequencies lower than that of the first choromospheric mode whose angular frequency lies at about $2.4\ 10^{-2}$. For higher frequencies, our values

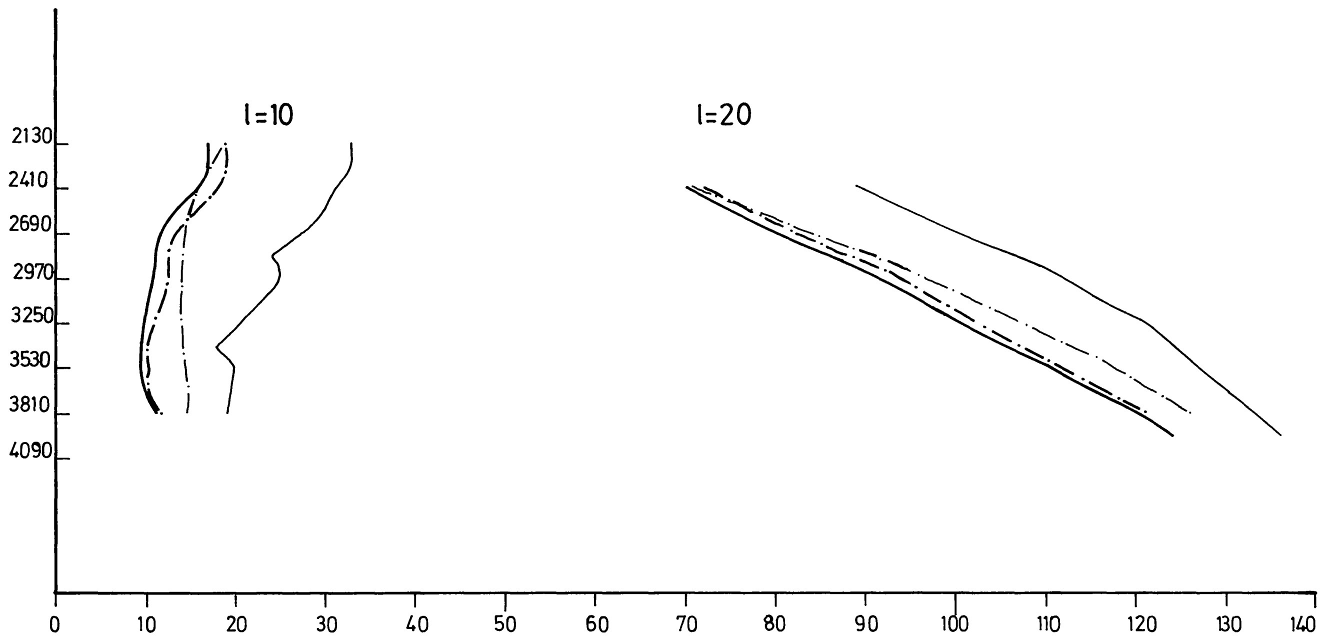

Fig. 4. Folded spectrum for $l=10$ and 20. The heavy full lines correspond to our results for $Z=0.018$; the heavy dot-dashed curves to our results for $Z=0.02$, the dot-dashed curves to Ulrich and Rhodes' (1983) results and the full lines to Duvall and Harvey's (1983) observations

Table 6. Angular frequencies of high l p modes for $Z=0.018$

Mode	$l=200$	$l=400$	$l=600$	$l=800$
f	$8.956\ 10^{-3}$	$1.260\ 10^{-2}$	$1.539\ 10^{-2}$	$1.775\ 10^{-2}$
p_1	$1.242\ 10^{-2}$	$1.616\ 10^{-2}$	$1.921\ 10^{-2}$	$2.205\ 10^{-2}$
p_2	$1.508\ 10^{-2}$	$1.969\ 10^{-2}$	$2.327\ 10^{-2}$	$2.654\ 10^{-2}$
p_3	$1.749\ 10^{-2}$	$2.316\ 10^{-2}$	$2.739\ 10^{-2}$	
p_4	$1.987\ 10^{-2}$	$2.624\ 10^{-2}$		
p_5	$2.209\ 10^{-2}$	$2.924\ 10^{-2}$		
p_6	$2.421\ 10^{-2}$	$3.206\ 10^{-2}$		
p_7	$2.619\ 10^{-2}$			
p_8	$2.818\ 10^{-2}$			

Table 7. Angular frequencies for high l p modes for $Z=0.02$

Mode	$l=200$	$l=400$	$l=600$	$l=800$
f	$8.961\ 10^{-3}$	$1.261\ 10^{-2}$	$1.540\ 10^{-2}$	$1.775\ 10^{-2}$
p_1	$1.243\ 10^{-2}$	$1.614\ 10^{-2}$	$1.916\ 10^{-2}$	$2.195\ 10^{-2}$
p_2	$1.506\ 10^{-2}$	$1.963\ 10^{-2}$	$2.300\ 10^{-2}$	$2.647\ 10^{-2}$
p_3	$1.746\ 10^{-2}$	$2.308\ 10^{-2}$	$2.727\ 10^{-2}$	$3.089\ 10^{-2}$
p_4	$1.982\ 10^{-2}$	$2.619\ 10^{-2}$	$3.120\ 10^{-2}$	
p_5	$2.204\ 10^{-2}$	$2.914\ 10^{-2}$		
p_6	$2.417\ 10^{-2}$	$3.194\ 10^{-2}$		
p_7	$2.618\ 10^{-2}$			
p_8	$2.806\ 10^{-2}$			
p_9	$2.994\ 10^{-2}$			

become up to 3 % too large. This can be understood in terms of the convective zone depth, smaller in our models than in these of Shibahashi et al. (1983). Our value is also smaller than the depth of 2.3 10^5 km required to obtain a good fitting (see Berthomieu et al., 1980).

6. Conclusions

We have computed frequencies for low ($0<l<5$) and intermediate ($l=10$ and 20) l values with an accuracy better than 2 μHz. Deviations larger than 2 μHz are found between our values and Ulrich and Rhodes's and Shibahashi et al.'s. They can only be explained by differences in the three sets of standard solar models. The origin of these differences should be clarified. The theoretical frequencies computed by the three groups deviate from the observations by several times the sum of the theoretical and observational errors and the shape of the folded spectra cannot be reproduced.

For high l the frequencies are accurate to better than 10^{-3} but deviate from the observations by 1 – 3 %. This is not surprising as it is well known that a convective zone depth of the order of 2.3 10^5 km is necessary to reproduce the observations while in our models, this depth reaches only 1.75 10^5 km.

As in the whole range of l values, all standard solar models fail to reproduce the observations, it can be concluded that something is missing in the physics of the Sun's evolution. The high l values as well as the results of Scuflaire et al. (1980, 1982) and Gabriel et al. (1982) suggest that the convective zone should be much deeper than found in standard models. In fact, Shibahashi et al.'s models have the deepest convective envelopes among all standard models and do provide the best fit so far obtained. Therefore the physical improvement to bring into the models must have the effect of deepening the convective envelope.

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