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Research Note

On the Relation Between True and Apparent Flattenings of Elliptical Galaxies

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Summary. The integral equation relating the distribution of shapes of elliptical galaxies to the distribution of shapes of their images takes a simpler form when the eccentricity is used instead of the axes ratio. This equation may be reduced to Abel's equation and its solution may be expressed analytically, which has remained unnoticed up to now.

Key words: elliptical galaxies

If we assume that elliptical galaxies are oblate spheroids, the Hubble's relation (Hubble, 1926; ten Bruggencate, 1930) relates the true flattening of the galaxy and the flattening of its image. This relation assumes a simple form if it is expressed in terms of e the eccentricity of a meridian section of the galaxy and e' the eccentricity of its image:

$$e' = e \sin i, \tag{1}$$

where i is the angle between the line of sight and the revolution axis of the galaxy.

Let $f(e)\,de$ be the fraction of elliptical galaxies with eccentricities in the interval $[e, e+de]$ and let $f'(e')$ the distribution function similarly defined for the eccentricities of the images. Assuming random orientation of the revolution axes, both functions are related by the integral equation

$$\int_{e'}^{1} \frac{e' f(e)\,de}{e(e^2 - e'^2)^{1/2}} = f'(e'). \tag{2}$$

This equation is equivalent to Eq. (5) of Sandage et al. (1970) who used the axis ratios rather than eccentricities. The problem of deducing the distribution of e from the observed distribution of e' is formally the same as the deduction of the distribution of the true equatorial velocity v of rotating stars from the apparent velocity v';

$$v' = v \sin i.$$

This latter problem has been solved by Chandrasekhar and Münch (1950). There are here a few differences due to the fact that the eccentricities contrary to velocities vary only in the range [0, 1]. Let x, y, F, and G be defined by

$$x = 1 - e^2$$

$$y = 1 - e'^2$$

$$F(x) = f(e)/e^2$$

$$G(y) = 2f'(e')/e'.$$

We get

$$\int_0^y \frac{F(x)\,dx}{(y-x)^{1/2}} = G(y).$$

This equation is Abel's equation. It is discussed in Goursat (1917) and a more general form of it in Whittaker and Watson (1950). Its solution is

$$F(x) = \frac{1}{\pi}\frac{d}{dx}\int_0^x \frac{G(y)\,dy}{(x-y)^{1/2}}.$$

We finally obtain

$$f(e) = -\frac{2e}{\pi}\frac{d}{de}\int_e^1 \frac{f'(e')\,de'}{(e'^2 - e^2)^{1/2}}. \tag{3}$$

This solution requires the derivation of a function affected by observational errors and statistical errors due to the sampling. This can lead to misleading results. This remark justifies the use of numerical methods: Sandage et al. (1970) assume a given form with unknown parameters for the unknown function whereas Noerdlinger (1979) and Binney and de Vaucouleurs (1981) use the method described by Lucy (1974). Whereas expression (3) may be of little use, we suggest the use of eccentricities rather than axes ratios as the kernel of Eq. (2) is simpler than the kernel of the equation written by Sandage et al. (1970).

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