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The Solar Structure and the Low *l* Five-minute Oscillation. I

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Summary. The parameters X, Z, and l/H_p have been obtained by fitting the theoretical frequencies with the values observed by Claverie et al. (1981).

A normal chemical composition is found. To get accurate values for X and Z, the ratio Z/X must be imposed. They are then in good agreement with observations. The mixing length obtained is larger than predicted by standard solar models. The observed rotational splitting favours slow or moderately fast internal rotation laws compatible with Hill and Stebbins oblateness measurements.

Key words: solar oscillations – rotational splitting – solar oblateness

I. Introduction

Recently Claverie et al. (1981) published a new and more accurate list of 33 solar oscillation frequencies for spherical harmonic degree l equal to 0, 1, and 2 in the 5 min range. For the first time, they also succeeded in measuring the rotational splitting of the lines. This result provides the strongest constraint put on possible solar rotation laws known to date.

We present here the results of the comparison of these observations with the predictions of theoretical solar models.

Models similar to those used in Scuflaire et al. (1981) were computed for 3 chemical compositions ($X=0.7417$, $Z=0.02$), ($X=0.7417$, $Z=0.018$) and ($X=0.76$, $Z=0.02$). For each composition models were computed for 5 values of the ratio fo the mixing length to the pressure scale height l/H_p: 1.85, 2.4, 3, 4, and 5. The first chemical composition and $l/H_p=1.85$ corresponds to our standard solar model. More details concerning the models are given in Scuflaire et al. (1982). It is clear that this set of models do not represent standard solar models. However as the standard model fails to predict the neutrino flux, as well as to fit the high l 5 min oscillations [fitting requires a larger l/H_p see Rhodes et al. (1977, 1980), Berthomieu et al. (1980)] and the low l 5 min oscillations, it is reasonable to think that the inner sun is not accurately represented by the standard evolution. The method used here gives a first approach to obtain independently the parameters X, Z, and l/H_p. Of course we will have to verify that a "reasonable" core model can be adjusted to our best solution.

Eigenfrequencies have been computed for these 15 models for $0 \leqq l \leqq 4$ and orders of the overtone n varying between 10 and 30.

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II. Comparison of the Frequencies

The agreement between the observed frequencies and those computed for standard solar models is fairly good. It is good enough compared to the spacing between modes to allow an unambiguous identification of the observed modes. However our theoretical frequencies are always too small. The deviation varies from a few to about 20 μHz and systematically it increases with l while it decreases as n grows. This is also found by Shibahashi and Osaki (1981).

As the derivaties of the eigenfrequencies with respect to Z, X, and l/H_p are respectively of the order of 10^3, 10^2, and 10 it appears possible to reduce the discrepancy below the sum of the observational and theoretical errors by adjustment of the parameters X, Z, and l/H_p.

It is shown in Scuflaire et al. (1982) that the computed eigenfrequencies depend very non linearly on the mixing length. Therefore the fitting was done using a least square method linear for X and Z and non linear with respect to l/H_p. The frequencies and their 3 derivatives were interpolated with respect to l/H_p using the Lagrange formula. Two fittings have been done, the first with 3 parameters, the second with an imposed value for the Z/X ratio of 0.025. This value was chosen between these given by Grevesse (1981) $Z/X=0.0259$, Pagel (1979), $Z/X=0.026$, and Cameron (1980) $Z/X=0.02448$. The results are given in Table 1 where σ is the mean quadratic deviation given in μHz. Only one frequency ($l=0$, $n=17$) deviates from the observed value by more than 1.5σ.

Table 1 shows that we obtain normal chemical composition. With 3 parameters, the value of Z is however somewhat high. The values of the mixing length are definitively larger than the value of the standard solar models. This confirms the result obtained previously by Scuflaire et al. (1980) and the value quoted from the high l 5 min oscillations studies.

The mean quadratic deviations are remarkably small, probably not larger than the sum of the observational and theoretical errors. It is also hardly larger for the fitting with two parameters. This indicates that these high n quasi-asymptotic modes contain

Table 1. Parameters Z, X, and l/H_p and variance σ given by fitting of the observed and theoretical frequencies

	Z	X	l/H_p	σ
$Z/X=0.025$	0.0221	0.738	2.945	1.84
	0.0194	0.774	2.901	2.02

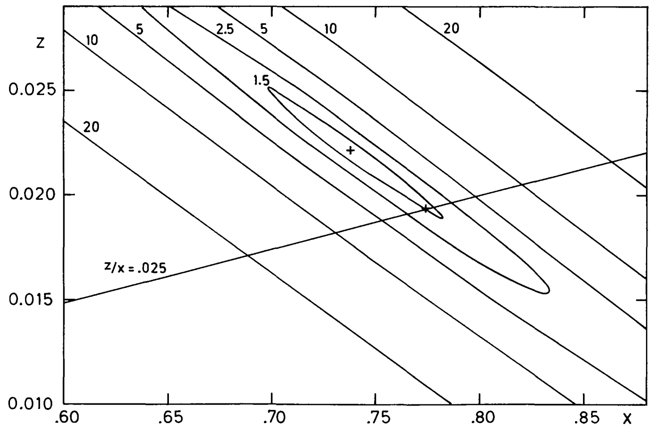

Fig. 1. Curves of equal average distance between observed and theoretical frequencies in terms of X and Z for a mixing length of 2.9. The total error in both one observed and one theoretical frequency is supposed equal to 1 μHz

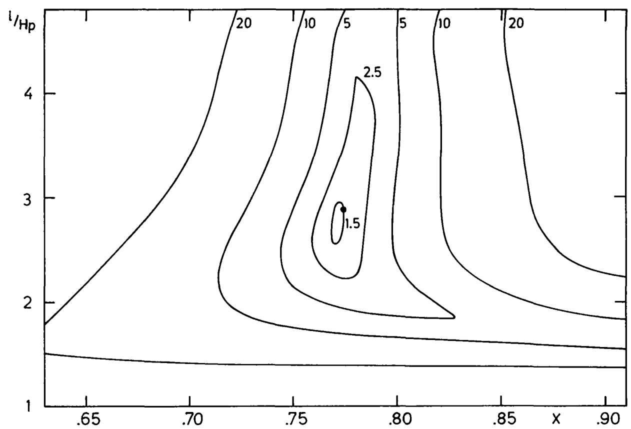

Fig. 2. Same as Fig. 1 but in term of X and l/H_p for $Z/X=0.025$

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only a small amount of information. The asymptotic theory (see Tassoul, 1980) shows that the eigenfrequency for high order overtones depends, to the first order, only of the average polytropic index of the outer layers and of $\int_0^R 1/c\,dr$, c being the sound speed. This implies that only two accurate parameters are easily obtained from the data. Additional informations being connected with higher order terms in the asymptotic theory, they will be obtained only if a very high accuracy is achieved both in the observational and on the theoretical sides (Remember that for any solar model the theoretical frequencies depart by at most 1% from the observations. To illustrate this further and to show the significance of the fitting we have computed the mean distance between the observed and the theoretical frequencies, supposing a total error of 1 μHz, in terms of X, Z, and l/H_p. Figure 1 gives the curves of equal distances for $l/H_p=2.9$. Figure 2 gives the same curves obtained imposing the condition $Z/X=0.025$. The equal distances curves are so elongated in the (X, Z) plane that, even if one considers the curve $d=1.5$ μHz, we cannot deduce any new significant value for Z. We can only see that X and Z are related by the linear law

$$Z=0.0775-0.075X\,.$$

It is only when the ratio Z/X is imposed that we obtain a chemical composition which is meaningful. It is surprising to see how close it is from Cameron's ($X=0.772$, $Z=0.0189$) and it is also in good agreement with Pagel ($X=0.76$, $Z=0.02$) and Grevesse ($X=0.78$, $Z=0.02$) results. It corresponds however to a X value larger than required by a standard solar model. It also favours a N_{He}/N_H ratio of the order of 0.06 rather than values around 0.1.

Both fittings give values for l/H_p of about 2.9. Figure 2 shows that the mean distance increases quickly enough to exclude values of the order of 1.85 given by the standard solar model.

That value $l/H_p=2.9$ corresponds to a depth of the convective zone of the order of $2.2\,10^5$ km, and a temperature at its base between 2.5 and $2.6\,10^6$K.

III. The Rotational Splitting

The average rotational splitting is 0.77 μHz in an inertial frame of reference (Isaak, 1981). Using the equatorial solar rotation given by Scherrer et al. (1980), this gives a dimensionless parameter $S=(2\pi\Delta v/m\Omega_s)=1.66$ where Ω_s is the surface angular velocity and m the azymuthal spherical harmonic index.

Two kinds of observations provide us with information concerning the internal rotation of the sun. The oblateness related to the centrifugal force is proportional to Ω^2. The splitting due to the Coriolis force is proportional to Ω. It is therefore expected to provide a much more sensitive test for possible rotational laws than the oblateness on the condition that the amplitude of the eigenfunctions remains large enough inside the sun. We will verify that this is indeed the case for the low l 5 min oscillations. We will also deduce simple rotational law compatible with the observed splitting.

To do this we will suppose that Ω is a function of the radius only. This hypothesis might be questioned though Endal and Sofia (1980) have given arguments to show that this is a very good approximation. It has however the advantage of being easier to handle numerically and it is anyway sufficient for a first approach.

The rotational splitting for non solid rotation has been studied by Hansen et al. (1977). To the first order in Ω and in an inertial frame of reference it writes if $\Omega=\Omega(r)$

$$S=\frac{\int_0^M \frac{\Omega}{\Omega_s}\left\{\frac{\chi}{\sigma^2 r}\left(2\delta r+\frac{\chi}{\sigma^2 r}\right)-\left[\delta r^2+l(l+1)\left(\frac{\chi}{\sigma^2 r}\right)^2\right]\right\}dm}{\int_0^M\left[\delta r^2+l(l+1)\left(\frac{\chi}{\sigma^2 r}\right)^2\right]dm}\,. \quad (1)$$

The notations are the same as in Ledoux and Walraven (1958).

For a solid rotation, one often writes (1) in the form

$$S=-(1-C_{ln})\,.$$

For the modes with $l=1$ and 2 and $15<n<31$ $C_{ln}<10^{-2}$.

Figures 3 gives the integral of the numerator of (1) from 0 to r with $\Omega/\Omega_s=1$ and a denominator normalized to unity for the mode $l=2$, $n=21$. It allows the reader to evaluate the rotational splitting when Ω is a sum of step functions. It also shows the contribution of each layer to the splitting.

The theory of solar oblateness can be found in Roxburg (1964), Goldreich and Schubert (1968), and Fricke (1969). It is obtained from the surface value of the solution of a second order non homogeneous differential equation $L_2(y)=-f$ satisfying the surface condition $y'+3y=0$.

The operator L_2 has a regular singular point at the center and is independent of the rotation law which appears only in f.

When one works out the solution of this problem by the Green's functions method, it turns out that the surface value of the solution y_s is simply given by

$$y_s = \frac{\int_0^1 x f y_1 dx}{(3y_1 + y_1')_s},$$

where y_1 is the solution of $L_2(y)=0$ regular at the center.

This solution, which does not seem to have been found before, is very simple to compute and allows easily to evaluate the oblateness for lots of rotational laws.

The best attempt to find the internal solar rotation law was made by Endal and Sofia (1980).

We first consider the law they obtained in their case C. From their Fig. 6, it can be roughly approximated by the law

$$\Omega = 4.26\,\Omega_- \exp[-2.303(x-0.37)] \quad \text{for} \quad x<0.37, \tag{2}$$

$$\Omega = \Omega_- \exp[-3.921(x-0.74)] \quad \text{for} \quad 0.37<x<0.74, \tag{3}$$

$$\Omega = \Omega_s \quad \text{for} \quad x>0.74, \tag{4}$$

$$\Omega_- = 4.64\,\Omega_s. \tag{5}$$

It leads to a splitting $S \simeq 7.1$ and an oblateness $\varepsilon = 5.6\,10^{-5}$. Though 15% smaller than that of Endal and Sofia, our value for ε may be considered in agreement with theirs as our rotation law is only a crude approximation of the one they find. The predicted splitting is more than 4 times too large and corroborates Endal and Sofia's conclusion that their internal rotation is too fast. If we now replace (5) by $\Omega_- = \Omega_s$, we obtain $S=2$ which is still 20% too large and $\varepsilon = 1.31\,10^{-5}$ compatible with Hill and Stebbins observation. Endal and Sofia suggested that "some or all the circulation region ($0.74>x>0.37$) should be rotating at nearly the same rate as the convective zone". Therefore we replaced (3) by $\Omega = \Omega_s$. We get $S=4.67$ and $\varepsilon = 2.95\,10^{-5}$ which are much too large. However if we further replace (5) by $\Omega_- = \Omega_s$ we obtain

$$1.60 < S < 1.77$$

and $\varepsilon = 1.26\,10^{-5}$ in good agreement with the observations.

We have also computed several rotation laws which reproduce the observed splitting. Continuous linear laws and quadratic ones of the form $\Omega = C_1 + C_2\,(x-x_0)^2$ have been considered. The results are summarized in Table 2 where Ω_c is the central angular velocity. All the linear laws dynamically unstable while the quadratic ones are always stable. It turns out that the ratio Ω_c/Ω_s is certainly lower than 30. All but one rotation laws compatible with the observed splitting produce an oblateness differing by less than 1σ from the value of $\varepsilon = (9.6 \pm 6.5)\,10^{-6}$ given by Hill and Stebbins (1975). They are all incompatible with Dicke and Goldenberg's (1968) result $\varepsilon = (5.0 \pm 0.7)\,10^{-5}$.

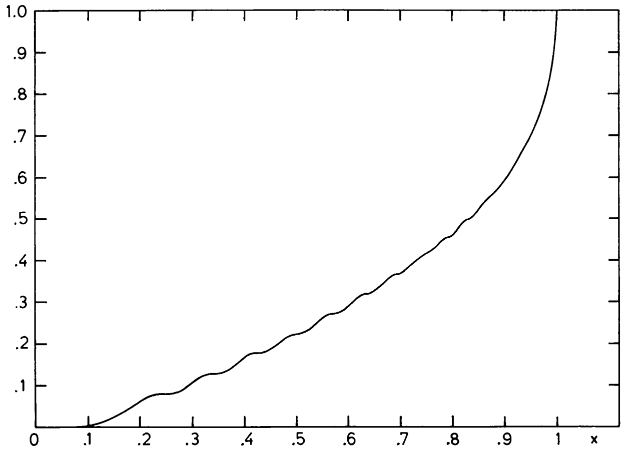

Fig. 3. Integral of the numerator of Eq. (1) in term of $x=r/R$ for $\Omega/\Omega_s = 1$

Table 2. Rotation law fitting the observed splitting and the associated oblateness

Rotation law		Ω_c/Ω_s	$\varepsilon \times 10^5$
Linear for $x<0.74$, $\Omega=\Omega_s$	$x>0.74$	4.9–5.5	<1.24
Linear for $x<0.74$, $\Omega=\Omega_s/x^2$	$x>0.74$	3.1–3.3	<1.33
Linear for $x<0.37$, $\Omega=\Omega_s$	$x>0.37$	12–15	<1.22
Linear for $x<0.37$, $\Omega=\frac{\Omega_s}{(0.74)^2}$	$0.37<x<0.74$	5–6.3	<1.21
$\Omega=\Omega_s/x^2$	$x>0.74$		
Quadratic for $x<0.74$, $\Omega=\Omega_s$	$x>0.74$	6.6–8.8	<1.12
Quadratic for $x<0.74$, $\Omega=\frac{\Omega_s}{x^2}$	$x>0.74$	4.5–4.6	<1.18
Quadratic for $x<0.37$, $\Omega=\Omega_s$	$x>0.37$	20.–28	<1.43
Quadratic for $x<0.37$, $\Omega=\frac{\Omega_s}{(0.74)^2}$	$0.37<x<0.74$	13.–16.	<2.07
$\Omega=\frac{\Omega_s}{x^2}$	$x>0.74$		

Conclusion

The observational accuracy now achieved by Claverie et al. in the measurement of the frequencies of low l solar oscillations allows a reliable determination of the solar chemical parameters X and Z and mixing length, provided the ratio Z/X is given. The chemical composition then obtained is in very good agreement with these given by Cameron (1980), Pagel (1979), and Grevesse (1981) and favour a N_{He}/N_H ratio of the order of 0.06. The mixing length is however larger than predicted by standard solar models as already found from the high l 5 min, oscillations. A higher accuracy should be required both on the observed and theoretical frequencies in order to deduce more information.

The measurement of the rotational splitting provides stronger constraints on possible rotational laws than the solar oblateness. All laws compatible with the splitting predict an oblateness in good agreement with Hill and Stebbins value but in disagreement with Dicke and Goldenberg's result. The rotational splitting favours slow or moderately fast internal rotation and supports Endal and Sofia's suggestion that the circulation region is rotating with nearly the same velocity as the convective envelope.

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