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# The Solar Structure and the Five-minute Oscillation

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**Summary.** A sequence of solar envelope models with convective zones of variable thickness has been computed. The corresponding theoretical eigenfrequencies of radial and nonradial modes were then compared with recent observations. This favours a model with a convective zone containing 7.5% of the mass and extending over 34% of the radius. Such a convective zone is much deeper than in the standard solar model.

**Key words:** solar oscillations

## 1. Introduction

The five-minute solar oscillation has been the object of detailed studies.

The observations of Deubner (1975), Rhodes et al. (1977), Deubner et al. (1979) have simultaneously resolved the motion of the solar surface in frequency and in horizontal wave-number. They have brought out the $f$-mode and the first few $p$-modes of large spherical harmonics degree $\ell$ (typically a few hundreds). Modes of this type have been computed for solar models by Ulrich (1970), Ando and Osaki (1975, 1977), Ulrich and Rhodes (1977), Lubow et al. (1980), Berthomieu et al. (1980), and Rhodes and Ulrich (1980). They have found that the theoretical frequencies are sensitive mainly to the depth of the convective zone and that these observations are better accounted for with a convective zone deeper than was previously thought.

Claverie et al. (1979, 1980) succeeded in observing oscillations near five minutes in integral sunlight. They interpreted them as low $\ell$, high overtones pulsations. Christensen-Dalsgaard and Gough (1980) showed that they must be modes with $\ell$ values lower than 4.

We have computed a sequence of solar envelope models. They are described in Sect. 2. The eigenfrequencies are computed and compared with those observed by Claverie et al. (1980) for low $\ell$ values and with those of Deubner et al. (1979) for high $\ell$ values. The consequences of these comparisons for the solar structure are then discussed.

## 2. The Models

A sequence of evolution has been computed with the Henyey code (Henyey et al., 1964, 1965) for a solar model with an initial composition given by $X=0.7417$, $Z=0.02$. The equation of state is given by Bodenheimer et al. (1965). The nuclear reaction rates are those of Fowler et al. (1967). For the opacity coefficient, we took the values of Cox and Stewart (1970), modified at low temperatures according to Alexander (1975) to take into account the effect of molecules. The model corresponding to the solar age ($4.5\,10^9$ yr), model 1, has the correct (solar) radius ($R_\odot = 6.96\,10^{10}$ cm) and luminosity ($L_\odot = 3.82\,10^{33}\,\mathrm{erg\,s^{-1}}$) when a value of 1.85 is chosen for $\ell/H$, the ratio of the mixing-length to the pressure scale height. We were particularly careful in the computation of the external layers and the model includes a model of chromosphere whose temperature profile is given by Vernazza et al. (1973). The model of chromosphere extends up to the discontinuity of temperature located at 2400 km above the photosphere. We assumed that the pressure vanishes at this point. From the bottom of the convective zone up to this point the model includes about two hundred points. As most evolution codes do, the model has been broken into two parts for the computation: an interior and an envelope. The mesh size is smaller in the envelope than in the interior. At the transition between these two parts (which corresponds to a temperature of about $3\,10^5$ K) we required a relative precision of 0.001 on the fit.

The other models considered here (models 2 to 5) are solar envelope models comprising 80% of the mass. They are obtained by an inward integration alone, they do not fit an interior model. The temperature at their basis is of the order of $10^7$ K. They all have the solar radius and luminosity and the same chemical composition as our model 1. They differ by the thickness of the convective zone, corresponding to different values of $\ell/H$. We shall not discuss here the difficult problem of which type of interior can match these envelope models. Table 1 gives some

**Table 1.** Some characteristics of the models. $\ell/H$ is the ratio of the mixing-length in the convective zone to the pressure scale height, $r$, $\varrho$, and $T$ the radius, the density and the temperature at the bottom of the convective zone, $m_c$ the mass of the convective zone and $\tau$ the life-time of $Li^7$ in the convective zone

| Model no. | $\ell/H$ | $r$ ($R_\odot$) | $m_c$ ($M_\odot$) | $\varrho$ ($\mathrm{g\,cm^{-3}}$) | $T$ ($10^6$ K) | $\tau$ ($10^9$ yr) |
|---|---|---|---|---|---|---|
| 1 | 1.85 | 0.744 | 0.014 | 0.106 | 1.88 | 91,200 |
| 2 | 2.4 | 0.704 | 0.031 | 0.239 | 2.30 | 421 |
| 3 | 3 | 0.674 | 0.053 | 0.396 | 2.63 | 16.3 |
| 4 | 4 | 0.649 | 0.084 | 0.613 | 2.92 | 1.36 |
| 5 | 5 | 0.633 | 0.107 | 0.781 | 3.10 | 0.342 |
| $X$ | 3.7 | 0.66 | 0.075 | 0.548 | 2.83 | 2.74 |

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**Table 2.** The frequencies $\nu$(mHz) of modes of order $n$ ($p_n$) of low $\ell$ values are given for model 1 in the range 2.3 to 4 mHz

| $n$ | $\ell=0$ | $\ell=1$ | $\ell=2$ | $\ell=3$ |
|---|---|---|---|---|
| 15 | | | 2.3365 | 2.3888 |
| 16 | 2.3524 | 2.4139 | 2.4706 | 2.5231 |
| 17 | 2.4865 | 2.5480 | 2.6052 | 2.6584 |
| 18 | 2.6213 | 2.6832 | 2.7407 | 2.7941 |
| 19 | 2.7566 | 2.8187 | 2.8766 | 2.9302 |
| 20 | 2.8924 | 2.9545 | 3.0122 | 3.0663 |
| 21 | 3.0280 | 3.0904 | 3.1485 | 3.2028 |
| 22 | 3.1640 | 3.2266 | 3.2850 | 3.3398 |
| 23 | 3.3004 | 3.3634 | 3.4220 | 3.4771 |
| 24 | 3.4372 | 3.5003 | 3.5595 | 3.6149 |
| 25 | 3.5744 | 3.6378 | 3.6969 | 3.7528 |
| 26 | 3.7116 | 3.7754 | 3.8349 | 3.8908 |
| 27 | 3.8492 | 3.9131 | 3.9728 | |
| 28 | 3.9869 | | | |

**Table 3.** The frequencies $\nu$(mHz) of modes of order $n$ ($p_n$) of low $\ell$ values are given for model 5 in the range 2.3 to 4 mHz

| $n$ | $\ell=0$ | $\ell=1$ | $\ell=2$ | $\ell=3$ |
|---|---|---|---|---|
| 15 | | | 2.3486 | 2.4056 |
| 16 | 2.3552 | 2.4205 | 2.4825 | 2.5395 |
| 17 | 2.4899 | 2.5534 | 2.6162 | 2.6735 |
| 18 | 2.6245 | 2.6866 | 2.7499 | 2.8073 |
| 19 | 2.7591 | 2.8198 | 2.8833 | 2.9414 |
| 20 | 2.8935 | 2.9535 | 3.0167 | 3.0756 |
| 21 | 3.0277 | 3.0878 | 3.1502 | 3.2099 |
| 22 | 3.1619 | 3.2224 | 3.2842 | 3.3446 |
| 23 | 3.2962 | 3.3576 | 3.4183 | 3.4794 |
| 24 | 3.4302 | 3.4932 | 3.5530 | 3.6143 |
| 25 | 3.5645 | 3.6289 | 3.6882 | 3.7495 |
| 26 | 3.6991 | 3.7649 | 3.8239 | 3.8849 |
| 27 | 3.8339 | 3.9010 | 3.9600 | |
| 28 | 3.9692 | | | |

characteristics for the five models of the sequence. The life-time of lithium in the convective zone has been computed according to Fowler et al. (1975). It varies approximately as $T^{-24}$ ($T$ being the temperature at the bottom of the convective zone) and the ratio of the life-time to its local value at the bottom of the convective zone is approximately 14.1 for all models. Our values are in good agreement with those of Ulrich and Rhodes (1977).

### 3. Modes of Low $\ell$ Values

Modes of low $\ell$ values were computed as described in Boury et al. (1975). In order to have good eigenvalues and eigenfunctions we used a fourth order integration method with a variable stepsize independent of the mesh size of the model. The stepsize is chosen as the integration is proceeding in such a way to keep the discretization error small. Halving the stepsize, we have obtained, in a few test cases, an estimation of the error due to the discretization process. In our computation the relative error on the frequency amounts to a few $10^{-6}$. This result is achieved with about one thousand points. This error is quite negligible compared to the error arising from the lack of an interior model (for models 2 to 5), a problem we discuss just below.

**Table 4.** For all the models and for each value of $\ell$ the table gives the frequency $\nu$ (mHz) closest to 3 mHz and the mean separation $\Delta\nu$ ($\mu$Hz) between frequencies in the range 2.3 to 4 mHz

| Model no. | $\ell=0$ | $\ell=1$ | $\ell=2$ | $\ell=3$ |
|---|---|---|---|---|
| 1 | 3.0280 | 2.9545 | 3.0122 | 2.9302 |
| | 136.21 | 136.29 | 136.36 | 136.55 |
| 2 | 3.0326 | 2.9612 | 3.0205 | 2.9405 |
| | 135.88 | 135.71 | 135.91 | 136.10 |
| 3 | 3.0325 | 2.9608 | 3.0220 | 2.9433 |
| | 135.50 | 135.20 | 135.34 | 135.58 |
| 4 | 3.0300 | 2.9569 | 3.0198 | 2.9430 |
| | 134.91 | 134.75 | 134.64 | 134.90 |
| 5 | 3.0277 | 2.9535 | 3.0167 | 2.9414 |
| | 134.50 | 134.59 | 134.28 | 134.48 |

As models 2 to 5 are envelope models we met a difficulty in the application of the internal boundary condition. Applying a rigid boundary condition at the bottom of these models can lead to rather large relative errors on the frequencies which may amount to 4 or 5%, as was shown by a test on model 1. Then we replaced the missing interior by an artificial distribution of density given by

$$\varrho(r)=a+br^2,$$

where the parameters $a$ and $b$ are determined so that $\varrho$ and $m$ be continuous. (The continuity of the pressure automatically results from the inward integration of the hydrostatic equilibrium equation.) We have verified on model 1 that this process introduces relative errors on the frequencies at most of the order of $2\,10^{-4}$.

Christensen-Dalsgaard and Gough (1980) have shown that the discrete frequencies observed by Claverie et al. (1979, 1980) in the range 2.3 to 4 mHz correspond to modes with $\ell<4$ and fall into nearly degenerate groups satisfying

$$\nu_{\ell,n}\simeq\nu_{\ell+2,n-1},$$

where $\ell$ is the degree of the spherical harmonics and $n$ the order of the overtone. The approximate relation

$$\nu_{\ell+1,n}\simeq\tfrac{1}{2}(\nu_{\ell,n}+\nu_{\ell,n+1})$$

also holds for these modes. These relations are confirmed by our computations. Tables 2 and 3 give the computed frequencies for our extreme models. The twenty-five frequencies observed by Claverie et al. (1980) may be alternately identified with a mode of even $\ell$ ($n=16$ to 28 for $\ell=0$ or 15 to 27 for $\ell=2$) and a mode of odd $\ell$ ($n=16$ to 27 for $\ell=1$ or 15 to 26 for $\ell=3$). Therefore the observed frequencies fall into two spectra.

As the frequencies of these two spectra as well as the computed spectrum for a given $\ell$ are nearly equally spaced, each spectrum may be represented by two parameters, the value of one frequency $\nu$ (we have chosen the frequency closest to 3 mHz) and the mean separation $\Delta\nu$ between frequencies in the range from 2.3 to 4 mHz. Table 4 gives these parameters for all models. In Fig. 1 we have represented the two observed spectra with their uncertainties by the crosses and each continuous line represents the computed spectra of a given $\ell$ for the sequence of models. We notice that the mean separation between frequencies is rather sensitive to the

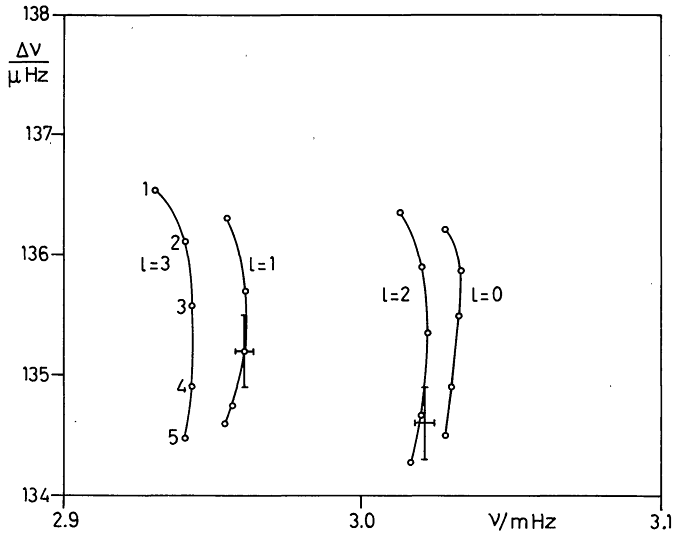


**Fig. 1.** $\nu$ and $\Delta\nu$ are respectively the frequency closest to 3 mHz and the mean separation between frequencies. The two crosses represent the observed spectra, respectively of odd and even values of $\ell$. The continuous curves represent for each value of $\ell$ the computed spectra for the sequence of models. For one of these curves the numbers label the models

**Table 5.** The frequencies $\nu$ (mHz) of modes of order $n$ of high $\ell$ values are given for model 1. The asterisk indicates the chromospheric mode

| $n$ | $\ell=200$ | $\ell=400$ | $\ell=600$ | $\ell=800$ |
|---|---|---|---|---|
| 0 | 1.4272 | 2.0077 | 2.4540 | 2.8304 |
| 1 | 1.9563 | 2.5672 | 3.0653 | 3.5192 |
| 2 | 2.3895 | 3.1082 | 3.6918 | 3.8880* |
| 3 | 2.7662 | 3.6489 | 3.8016* | 4.2360 |
| 4 | 3.1368 | 3.7264* | 4.3189 | 4.7562 |
| 5 | 3.4877 | 4.1611 | 4.6296 | 4.9366 |

**Table 6.** The frequencies $\nu$ (mHz) of modes of order $n$ of high $\ell$ values are given for model 5. The asterisk indicates the chromospheric mode

| $n$ | $\ell=200$ | $\ell=400$ | $\ell=600$ | $\ell=800$ |
|---|---|---|---|---|
| 0 | 1.4268 | 2.0086 | 2.4546 | 2.8302 |
| 1 | 1.9572 | 2.5733 | 3.0527 | 3.4939 |
| 2 | 2.3907 | 3.0893 | 3.6452 | 3.8869* |
| 3 | 2.7655 | 3.5932 | 3.7815* | 4.1512 |
| 4 | 3.1180 | 3.6820* | 4.2305 | 4.5597 |
| 5 | 3.4558 | 4.0804 | 4.4251 | 4.8061 |

thickness of the convective zone. Modes of $\ell=1$ and $\ell=2$ would match satisfactorily the observations for a model whose characteristics are given under the entry $X$ in Table 1.

## 4. Modes of High $\ell$ Values

Modes of high $\ell$ values (typically a few hundreds) were computed as described by Gabriel and Noels (1976), however the variables were chosen as in Gabriel and Scuflaire (1979). As $\delta r$ varies as $r^{\ell-1}$ in the central regions, the effect of the missing inner regions on the eigenfrequencies is completely negligible.

Solar oscillation modes of high $\ell$ values have been computed by a number of authors (Ulrich, 1970; Ando and Osaki, 1977; Lubow et al., 1980; Berthomieu et al., 1980). The comparisons with observations (Rhodes et al., 1977; Berthomieu et al., 1980; Rhodes and Ulrich, 1980) rule out a shallow convective zone: it must at least extend down to $0.7R_\odot$.

Our own computations (Tables 5 and 6) confirm the necessity of a deep convective zone. Figure 2 shows that our model with the deepest convective zone (model 5) agree better with the observations of Deubner et al. (1979) than our standard solar model (model 1).

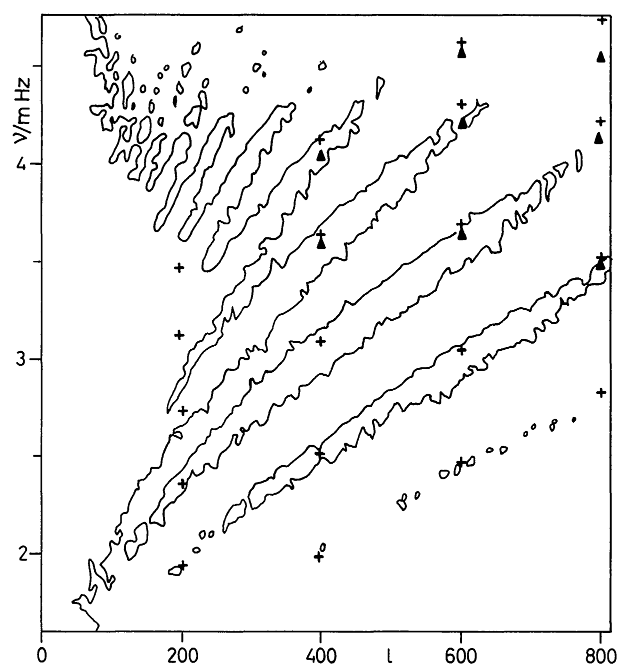


**Fig. 2.** The power contour in this $\ell$, $\nu$ diagram is taken from Deubner et al. (1979). The crosses indicate the computed modes of model 1. When they differ significantly from those of model 1 the frequencies of model 5 have been represented by triangles. The chromospheric mode has been omitted

## 5. Conclusion

As the observations of the five-minute oscillation are becoming more refined, the requirements imposed on the models of the external layers of the sun are getting more severe. We have shown that they can be interpreted with models having a normal solar composition, it is not necessary to resort to models with a low $Z$ in the interior as first suggested by Claverie et al. (1979) from the low $\ell$ observations. Both the low and high $\ell$ observations require a model with a deep convective envelope extending down to $0.66R_\odot$ and containing 7.5% of the mass. This result is compatible with the $Li^7$ abundance observed in the sun as its lifetime remains greater than $10^9$ yr.

The problem we have now to face is that the standard solar model requires a much thinner convective zone than found here. It is tempting to conclude that after the neutrinos observations, the solar oscillations show that something is wrong in the standard solar models.

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